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ECON INCORPORATED

A Distribution Benefits Model for Improved Information on Worldwide Crop Production

Volume II

Application to Various Crops

ECON INCORPORATED

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FINAL

A Distribution Benefits Model for Improved Information on Worldwide Crop Production

Volume II

Application to Various Crops

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NOTE OF TRANSMITTAL

This report is prepared for the National Aeronautics and Space Administration, Office of Applications, under Contract NASW-2558.

The methods developed in this report for estimating benefits of improved information are the best ECON is aware of at the time of writing. However, the subject is immensely complex, and it is possible that later work will improve on the methods and the accuracy of the results. ECON has maintained a conservative viewpoint on potential economic benefits of improved information, so that the estimates presented are more likely to be on the low side than the high side.

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ABSTRACT

ECON's distribution benefits model has been applied to worldwide distribution of corn, rye, oats, barley, soybeans, and sugar, and to domestic distribution of potatoes. The results indicate that a LANDSAT system with thematic mapper might produce benefits to the United States of about $119 million per year, due to more efficient distribution of these commodities. The benefits to the rest of the world have also been calculated, with a breakdown between trade benefits and those associated with internal use patterns. By far the greatest part of the estimated benefits are assigned to corn, with smaller benefits asigned to soybeans and the small grains (rye, oats, and barley). The methods of this study reveal no benefits to the United States in potato distribution, unless the LANDSAT system performs far better than currently anticipated. Potential United States benefits of improved sugar crops production information appear to be minor.

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1.0 INTRODUCTION

Volume I of this study gives the structure of an infinite horizon commodity distribution model which can be applied to specific commodities in various ways. In this volume, we report on the application of the model to sugar, potatoes, corn, soybeans, and small grains other than wheat.

The model describes the commodity distribution system as a control process, using time-dependent state variables and state transformations. Technical descriptions of this formulation are given in Volume I, in Chapters 3 and 4 of this volume, and in ECON's 1975 report, The Value of Domestic Production Information in Consumption Rate Determination for Wheat, Soybeans, and Small Grains. [2] In the 1975 report, the model was applied in a simple form using a single state variable and a one-year horizon. This same form is used in the present report for the case of potatoes. More than one state variable is required, however, for the application to the other crops considered, in which there is significant international trade involving the United States. With more than one state variable, the numerical calculations are more time-consuming, but the mathematical formalism is basically the same as for one state variable. Essentially the only difference is that various scalar quantities become vector or matrix quantities.

In going from the scalar model with a one-year horizon to the vector model with infinite horizon, there is a need for a substantially more complex procedure for solving the dynamic programming equations. In Volume I of this study, a procedure was presented which depended on value iteration to the point of convergence (a standard dynamic programming technique). Some improvements in this procedure have been incorporated in the work reported in this volume, leading to a substantially faster and more accurate solution of the equations.

The economic benefits model, based on the control process description of commodity distribution, requires as input a statistical description of the current crop information system and the improved system being evaluated. This description is called the supply information system model, and is discussed in Chapter 2. Three forms of an improved system are considered throughout this study. The first provides improved information on the producers other than the United States. The second provides information just on the United States, and the third provides a greater improvement than the first on the other producers, while providing the same improvement as the second on the United States.

The direct output of the dynamic programming section of the model is a table of value function coefficients. These coefficients can be multiplied by the variances describing a given supply information system to determine the net economic loss (relative to zero variance) associated with that information system. Then to compare one information system with another, we simply form the difference of the two values of the loss function.

Chapters 3 through 7 present the variations of the modeling technique and the numerical results for the individual crops considered. Chapter 8 contains the summary and conclusions of the study. An appendix is included, giving the detailed algorithm that was programmed in APL for the calculations.

2.0 SUPPLY INFORMATION SYSTEM MODEL

To assess the benefits of a new information system to be implemented some years in the future, we would like to have, for comparison, a quantitative description of the existing information system of that time. The assumption of this study is that the base case information system will perform just as the existing system is performing currently. Thus, we do our best to use historical data to assess the present quality of crop production information in the United States and in the rest of the world.

This approach presents several difficulties. One is that the historical data represent readily available published information alone. It is possible that published forecasts and estimates of production and other supply factors may not represent the best information that was available at the time of publication. Another difficulty is that publication schedules and methods of estimation change from time to time, and in many regions of the world, figures are published erratically. Because of these factors, time series long enough for valid statistical inference are often not available. Futher, the production/distribution systems themselves tend to change over time, so that past data do not necessarily indicate the current level of information system performance.

For these reasons, even a thorough and sophisticated analysis of existing public data often produces an imprecise and inconclusive description of information system performance. Therefore, some points in the analysis call for essentially judgemental inputs. To the extent possible, however, we procede as described in the next section.

2.1 Forecast Variances

We divide the marketing year for a crop in a given country or region into periods. In some cases, the periods are monthly. In other cases, they are bi-monthly or quarterly. We obtain from the available historical data a sequence of production forecasts (or estimates) from the earliest available through the end of the marketing year. When there is a single, authoritative source, such as the USDA Crop Reporting Board for the United States, or F.O. Licht's International Sugar Report for worldwide sugar, much of the production forecast data is taken directly from that source. In some cases, there is some information relevant to production before actual production estimates are published. This can come in the form of planting intentions or planted acreage estimates, or simply qualitative statements of expected increase or decrease from the previous year. When there are acreage estimates, they are multiplied by previous yield figures to produce a surrogate production forecast. Our procedure begins with the collection or construction, one way or another, of a matrix of production forecasts. Each row of the matrix refers to the crop for a given marketing year, and each column refers to a given publication date within or before the year. Data from up to 15 years are used.

Denote by F_{ij} the j th forecast for the i th year. Let n be the number of years of historical data, so that $i = 1, \ldots, n$, and let m be the number of separate estimates of each year's production, so that $j = 1, \ldots, m$. In general, there is a trend or other pattern in the final estimates, F_{im}, over the years of the sample. Thus, we expect that the best estimate available before publication of F_{i1} would be based on consideration of the patterns in $F_{1m}, F_{2m}, \ldots, F_{(i-1)m}$. We thus define F_{io}, for $i = 3, \ldots, n$, by the extrapolation of the least squares linear fit to $F_{1m}, F_{2m}, \ldots, F_{(i-1)m}$. For $i = 1, 2$, we define F_{io} as equal to F_{1m}. F_{io} represents an estimate of production for year i which considers no information specific to year i, but only the historical pattern known at that time.

From these data we wish to estimate the quantities $\sigma_0^2, \sigma_1^2, \ldots, \sigma_{m-1}^2$ for use in the benefits model. These provide a description of the quality of the information system producing the estimates F_{ij}. Recall that for $i > 0$, σ_i^2 is the variance, conditional on information available at time i, of the probability distribution of the production forecast due at time $i + 1$. σ_0^2 is the *a priori* variance of the earliest forecast of the year. Since the year to which $\sigma_0^2, \sigma_1^2, \ldots, \sigma_{m-1}^2$ apply is an unspecified future year, it is probably better to assume the production system will operate at that time on a scale close to the present one than to assume it is represented by an average over the n-year sample. Thus we form the normalized difference vectors

$$D_k = \begin{pmatrix} (F_{1k} - F_{1(k-1)}) \dfrac{F_{n(k-1)}}{F_{1(k-1)}} \\ \vdots \\ (F_{nk} - F_{n(k-1)}) \dfrac{F_{n(k-1)}}{F_{n(k-1)}} \end{pmatrix}$$

for each $k = 1, 2, \ldots, m$. The new forecast at a time $i + 1$ in the future year is then assumed to differ from the time-i forecast by a quantity ϕ_i having the variance of the elements of D_{i+1}. Accordingly, we set

$$\sigma_i^2 = \mathrm{Var}(D_{i+1}) \text{ for } i = 0, 1, \ldots, m - 1 .$$

We will refer to the vector

$$\Sigma^2 = (\sigma_0{}^2, \sigma_1{}^2, \ldots, \sigma_{m-1}{}^2)$$

as the "forecast variance" vector for the region (United States or rest of the world) under consideration.

Since our model can be used to simulate the operation of the crop distribution systems, we can compare the price variability for each commodity as predicted by the model with the actual price variability of particular historical periods. In some cases, when statistics on supply information are poor, this comparison can be used to calibrate the model. The vector Σ^2 can be adjusted until the price variability predicted by the model matches history. In other cases, when the supply information statistics are satisfactory, but demand elasticity estimates are undependable, the same calibration procedure can be used to estimate demand elasticities. The former procedure was used in the case of sugar in this study, and the latter in the case of potatoes.

3.0 POTATOES

3.1 Information Factors in Potato Markets

Figure 3.1 shows cash potato prices in the United States from 1913 to 1975. The fluctuations are striking in their frequency and severity. It is clear that if potatoes were cheaply and easily storable, and this price history prevailed, anyone could make a fortune trading potato contracts! Thus, the primary story told by the data of Figure 3.1 is that potatoes are not particularly storable. In fact, storage of potatoes in the United States is limited to the fall crop, grown in the northern states, primarily Maine, Washington, and Idaho. The storage of fall potatoes (about 80% of the total) through the winter and spring contributes to smoothing consumption within the (marketing) year, but there is no carryover of potatoes from year to year. Figure 3.2 gives potato prices through a period of three years.

Because of the perishability of potatoes, there is considerable uncertainty associated with the adequacy of the stored fall crop. Thus the market responds to information not only on production, but also on shrinkage and loss. It is worth noting that shrinkage and loss are related to factors that can be estimated at the same

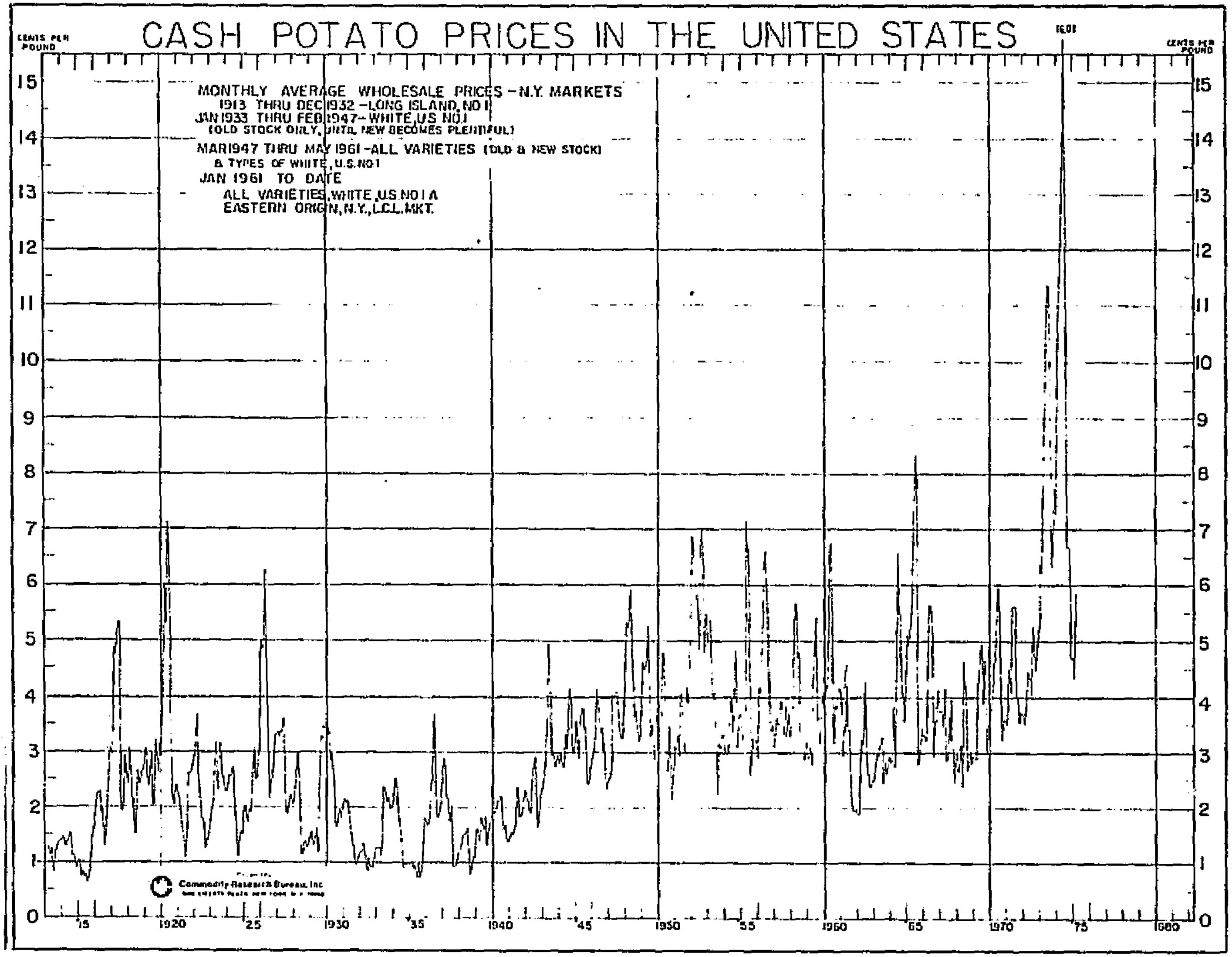

Figure 3.1 U.S. Cash Potato Prices

Source: 1975 Commodity Yearbook, page 277.

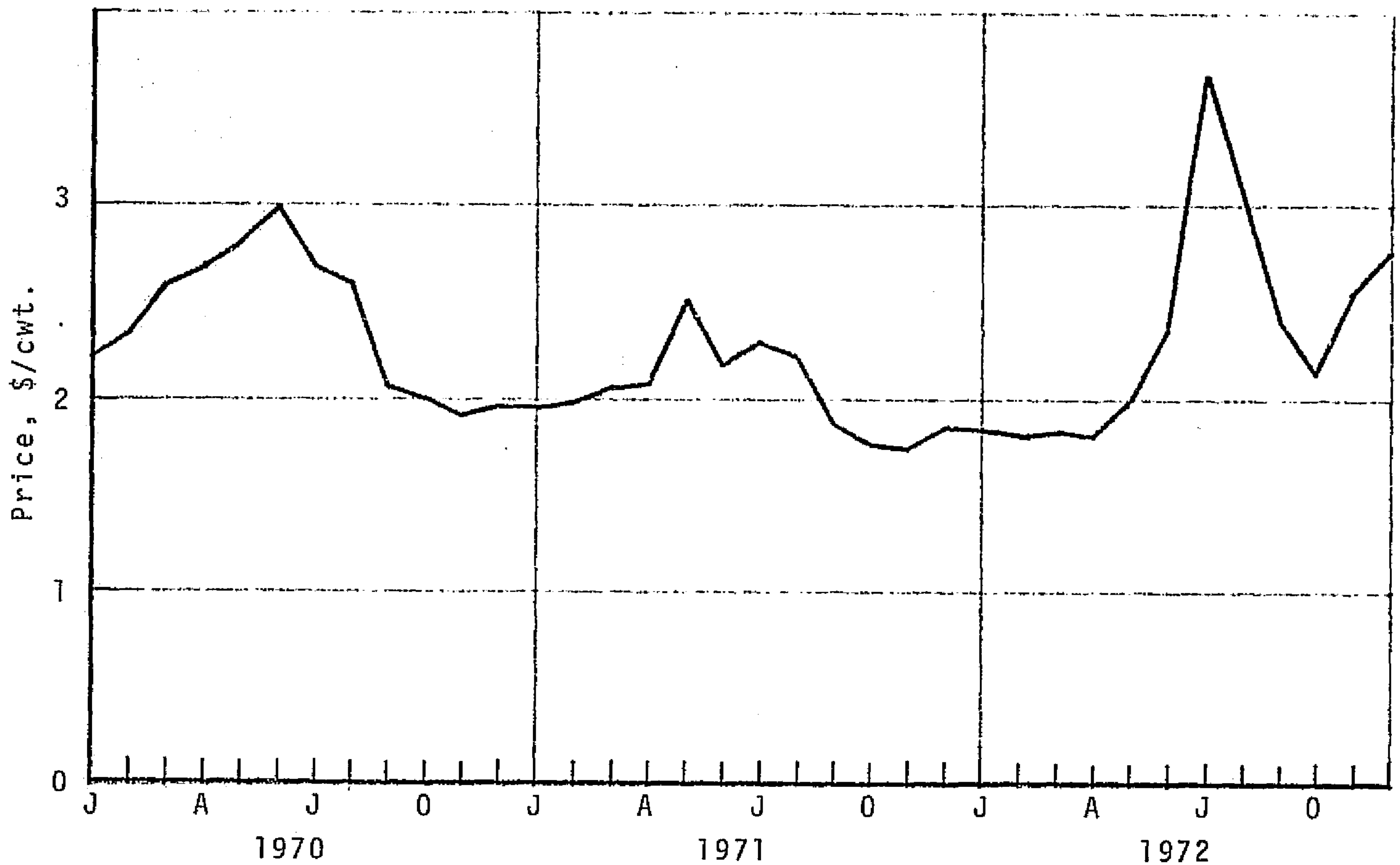

Figure 3.2 U.S. Cash Potato Prices, 1970-1972.

time as production. For instance, moisture content is critical, and this is determined largely by weather conditions during the growth period.

Most United States potatoes are grown for domestic consumption as food. Exports are ordinarily minor, amounting to near 1 percent of production. Consequently, United States potato prices reflect primarily supply and demand considerations in the United States alone.

3.2 Method of Applying Benefits Model to Potato Distribution

The form of the benefits model we use for information in potato production is quite simple. Since foreign trade is insignificant and there is no annual inventory carryover, we can model the process with a one-year horizon and a scalar state variable. The year is divided into monthly periods, and begins with harvest of the fall crop in September.

State and Decision Variables

The state variable, x_t, is the mean value, based on information at time t, of the remainder of the year's consumption. Since the year's consumption is the year's production minus shrinkage and loss, the state variable changes through time in response to new information on production, new information on shrinkage and loss, and as a result of consumption. The decision variable, denoted y, is the consumption for the period beginning at time t.

State Transformation

The state variable advances from time t to time $t + 1$ according to the formula

$$x_{t+1} = x_t - y + \phi_t ,$$

where ϕ_t represents new information on the year's total consumable supply, whether in the form of production estimate changes or shrinkage and loss estimate changes.

Value Functions

Economic value is generated in each period by the consumption y, according to the formula

$$F_t(y) = \alpha_t y^2 + \beta_t y ,$$

where α_t and β_t are obtained from the period-t demand function giving price p_t as a function of consumption y.

$$p_t = 2\alpha_t y + \beta_t .$$

In the case of potatoes, the parameters α_t and β_t change significantly from month to month, primarily because the processor demand is highly seasonal. Transportation and storage costs are quite significant for potatoes. In our model, the transportation costs are reflected in the demand parameters α_t and β_t. Storage costs have an extremely large fixed component, and except for interest on the capital invested in inventory, the costs are nearly independent of the length of the period of storage. Thus, the storage cost (excluding interest) is determined by the size of the fall crop, and is not influenced by the month-to-month storage/consumption decisions. Therefore, our value of information calculations deal only with the interest component of storage costs. Other storage costs are a constant to be subtracted from the total economic value of the crop, regardless of the information system under study.

The total value function for the dynamic programming formulation is denoted V_t. It is defined as follows. $V_t(x)$ is the mean value, based on information available at time t, of the remaining consumption for the marketing year, assuming the mean value of consumable supply for the rest of the year is x, and that the consumption pattern for the rest of the year is optimal.

<u>Optimality Principle</u>

The optimality principle of dynamic programming can be written as follows for this situation.

$$V_t(x) = \max_y \; [F_t(y) + \rho \overline{V_{t+1}(x - y + \varphi_t)}] \; . \qquad (1)$$

Here, ρ is the discount factor for one period. Its appearance here accounts for the interest costs associated with storage of potatoes. The horizontal bar denotes the mean with respect to the uncertainty in ϕ_t.

As usual, we represent V_t by the coefficients of its expansion.

$$V_t(x) = q_t x^2 + \ell_t x + k_t \; . \qquad (2)$$

Putting this expression in the functional equation (1) leads to the recursion relations

$$q_t = \frac{\alpha_t \rho q_{t+1}}{\rho q_{t+1} + \alpha_t} \; ,$$

$$\ell_t = \frac{\alpha_t \rho \ell_{t+1} + \beta_t \rho q_{t+1}}{\rho q_{t+1} + \alpha_t} \; , \qquad (3)$$

$$k_t = \rho k_{t+1} + \rho q_{t+1} \, \sigma_t^2 - \frac{(\beta_t - \rho \ell_{t+1})^2}{4(\rho q_{t+1} + \alpha_t)} \; ,$$

where σ_t^2 is the variance of ϕ_t; that is, $\sigma_t^2 = \overline{\phi_t^2}$ since $\overline{\phi_t} = 0$.

To solve the recursion relations, we observe that the boundary conditions are simply

$$q_{12} = \alpha_{12} ,$$

$$\ell_{12} = \beta_{12} ,$$

$$k_{12} = \alpha_{12}\sigma_{12}^2 ,$$

since the 12th period's consumption is of the remainder of the crop, and has mean value

$$V_{12}(x) = \alpha_{12}\overline{(x + \phi_{12})^2} + \beta_{12}\overline{(x + \phi)}$$

$$= \alpha_{12}x^2 + \alpha_{12}\sigma_{12}^2 + \beta_{12}x .$$

Now it is straightforward to determine $V_1(x)$ as a function of σ_1^2, ..., σ_{12}^2. Finally, the state value x at time 1 is itself stochastic, so the benefit estimates should be based on the mean value of $V_1(x)$ over the uncertainty in x. Letting σ_0^2 denote the variance in the time 1 estimate (September) of annual potato consumption, and μ_0 its mean, we obtain

$$\overline{V_1(x)} = q_1\mu_o^2 + \ell_1\mu_o + k_1 + q_1\sigma_o^2 \ .$$

The benefit of improved information is based on the change in this quantity in going from one information system to another.

3.3 Current Information System—Potatoes

3.3.1 Published Information

The Crop Reporting Board of the USDA publishes production estimates for United States potatoes in four seasonal groups. Fall potato production estimates now come in October, November, and December, although a September estimate was also published up through 1972. Winter potato production estimates are published in January, February, and March. An April estimate was published through 1972. Spring potato estimates are published in April, May, and June, and the summer crop estimates are published in July, August, and September.

Because of the perishablility of potatoes, the amount of shrinkage and loss of the fall crop in storage is an important uncertainty throughout the winter and spring. The Crop Reporting Board publishes stock reports in December, January, February, and March. By means of these, one can trace the total disappearance of the fall crop through

March 1. The market certainly responds to this information in estimating shrinkage and loss, but direct quantitative estimates of shrinkage and loss are not published with the stock reports. In our model, the stock reports are used to determine the average total disappearance pattern of the fall crop through time. We then model the uncertainty in shrinkage and loss by a stochastic process in which the shrinkage and loss estimates change uniformly over the stockholding period.

3.3.2 The Formal Model

The state variable in our dynamic programming formulation is x_t, the mean at time t of the remaining consumption for the year (September - August). Let π_t be the mean at time t of the year's production, and let λ_t be the mean at time t of the shrinkage and loss for the fall crop. For each t, we have

$$x_t = \pi_t - \lambda_t$$

We will discuss the development through time of π_t and x_t separately.

3.3.2.1 The Production Information Model

As discussed in Chapter 2, we base our analysis on production information quality on a matrix of "forecasts."

The first column of the forecast matrix represents n years of *a priori* forecasts; that is, of forecasts made without consideration of any information specific to the year at hand. The second column represents the best forecasts available at the beginning of consumption of the new crop. For United States potatoes, this time is Septmeber. Up to 1972, the Crop Reporting Board published a September forecast of fall potato production. but this forecast has been discontinued. We will use data only up to 1972 for those calculations that require the September forecast, but use data through 1975 for the remaining calculations. Thus, we assume in effect that the information represented by the old September forecasts is still present in the potato markets, although it is assembled privately rather than by the USDA.

Table 3.1 gives the published potato production estimates for the crop years 1960-1 through 1974-5. Table 3.2 gives the "forecast" matrix for all potatoes we contructed by summing the data of Table 3.1 and adjoining pre-season estimates as discussed in Chapter 2. The column labled "F_0" is obtained by extrapolation of previous years' production for all seasonal groups. "F_1" is obtained by adding extrapolated values of other seasonal groups to the published September estimate of fall potato production. "F_2", "F_3" and "F_4" are obtained similarly. The next three columns are abtained by adding the extrapolated values

Table 3.1 Potato Production Estimates Published by the USDA Crop Reporting Board, 1960-1974, millions of cwt.

Crop Year	Fall Potatoes					Winter Potatoes					Spring Potatoes					Summer Potatoes			
	Sep	Oct	Nov	Dec	Final	Jan	Feb	Mar	Apr	Final	Apr	May	Jun	Jul	Final	Jul	Aug	Sep	Final
1961-1	171.6	170.9	171.5	173.8	175.0					5.0	—			31.6	32.4	49.5	50.2	50.3	51.6
1961-2	192.2	193.7	198.4	201.5	204.6	4.2	4.2	4.3		4.2	—			24.1	21.7	46.5	47.1	47.0	46.4
1962-3	191.4	191.5	191.1	192.6	191.0	3.8	3.8		3.8	3.9	—	28.2	28.7	29.2	23.8	45.0	44.1	44.6	41.7
1963-4	190.3	194.0	195.7	195.9	197.3	3.6	3.7	3.7	3.6	3.7	—	23.3	23.7	24.48	19.7	39.9	39.5	38.9	39.0
1964-5	181.0	178.6	176.7	174.7	174.5	3.7	3.7	3.8	3.5	3.7	—	31.0	30.4	30.0	24.2	42.2	43.0	43.8	40.6
1965-6	209.5	216.0	215.5	213.4	216.8	5.4	5.1	5.1	5.0	5.1	—	31.9	32.7	32.0	25.9	46.6	53.6	43.4	43.1
1966-7	203.0	215.7	219.4	221.1	228.4	4.6	4.7	4.7	4.8	4.9	—	26.5	24.9	25.4	23.7	43.6	44.9	44.1	42.6
1967-8	223.5	229.5	231.1	232.1	231.7	3.8	4.1	3.9	3.8	3.9	—	24.8	25.0	25.1	20.5	44.0	44.1	44.0	43.9
1968-9	216.8	210.6	216.8	220.0	221.9	4.0	4.0	4.0	4.0	3.8	—	27.0	27.3	27.0	21.3	43.0	43.3	43.0	42.6
1969-0	231.9	231.5	231.2	233.6	239.5	3.7	3.6	3.6	3.5	3.6	—	24.5	24.8	25.6	21.2	41.4	41.3	41.7	42.7
1970-1	243.1	248.7	252.0	251.8	253.5	3.5	3.3	3.2	3.1	3.1	—	25.3	24.9	25.0	23.7	41.9	41.6	39.3	29.5
1971-2	246.9	249.3	250.8	249.9	266.7	2.7	2.5	2.5	2.4	2.3	—	21.1	21.1	21.4	21.1	34.5	35.6	35.7	37.0
1972-3	235.5	236.0	234.6	234.1	248.8	2.6	2.6	2.5	—	2.9	22.1	22.4	21.5	—	21.2	21.7	20.7	20.5	21.5
1973-4	—	252.7	253.8	252.0	253.9	2.5	2.6	2.8	—	2.8	22.7	23.1	23.6	—	24.3	23.9	24.4	24.6	25.2
1974-5	—	286.0	287.9	287.7	287.7	2.9	2.9	3.0	—	3.0	18.3	18.1	17.6	—		21.8	20.8	20.6	

Table 3.2 Potato Production Forecast Matrix for Use in Calculating Variances, millions of cwt.

Crop Year	F_0 (Extrapolated)	F_1 (Sep)	F_2 (Oct)	F_3 (Nov)	F_4 (Dec)	F_5 (Jan)	F_6 (Feb)	F_7 (Mar)	F_8 (Apr)	F_9 (May)	F_{10} (Jun)	F_{11} (Jul)	F_{12} (Aug)
1960-1	264	261	260	261	263	263	263	263	263	262	262	260	264
1961-2	264	281	283	287	291	290	290	290	290	282	282	276	277
1962-3	290	247	247	247	248	249	249	249	249	266	266	271	260
1963-4	263	247	250	252	252	253	253	253	253	259	259	264	260
1964-5	258	233	231	229	227	228	228	228	227	243	242	250	243
1965-6	243	266	273	272	270	273	272	272	272	285	286	297	291
1966-7	269	266	279	283	284	285	285	285	285	290	288	295	300
1967-8	287	289	295	296	297	297	297	297	297	299	300	305	300
1968-9	298	282	276	282	285	285	285	285	285	291	291	294	290
1969-0	299	297	296	296	298	298	298	298	298	302	302	304	307
1970-1	307	308	313	316	316	316	316	316	316	321	321	322	310
1971-2	315	310	311	314	313	312	312	312	312	311	311	308	327

of spring and summer production to the current estimates for fall and winter production. In the next three columns, extrapolated values of summer production are added to current estimates for the other seasonal groups. And in the final columns, the sum of current estimates are used. From the matrix of Table 3.2, the production information variances are calculated as described in Chapter 2. These variances are given in Table 3.4, together with the loss information variances discussed in the next section.

3.3.2.2 The Shrinkage and Loss Information Model

The disappearance from the fall potato crop due to shrinkage and loss has varied over recent years from about 10 million cwt. to about 28 million cwt.. Data on shrinkage and loss are available in USDA Statistical Bulletins No. 409 and 490, Potatoes and Sweetpotatoes, covering the years 1959 through 1969. Table 3.3 presents these figures in absolute terms and in terms of percentage of annual production.

Total disappearance (comsumption, shrinkage, and loss) of the fall crops proceeds quite smoothly through the months of the USDA stock reports, which are December through April. Typically, about 60% of the fall crop is in storage on December 1 and 20% on April 1 with a linear decline between these dates. For our modeling purposes we assume that the uncertainty facing the market in shrinkage and loss is

Table 3.3 Shrinkage and Loss of Potatoes 1959-1969

Year	Shrinkage and Loss, million cwt.	Total U.S. Production, million cwt.	Percent Shrinkage and Loss
1959	12.7	245.3	5.18
1960	13.0	257.1	5.06
1961	16.7	293.2	6.50
1962	13.6	264.8	5.14
1963	13.5	271.2	4.98
1964	10.3	241.1	4.27
1965	17.4	291.1	5.98
1966	28.2	307.2	9.18
1967	22.5	305.8	7.36
1968	19.0	295.4	6.43
1969	22.7	312.4	7.27
	Mean percent shrinkage and loss		6.12
	Standard deviation of percent shrinkage and loss		1.41

Source: USDA

Table 3.4 Supply Information Variances by Month, (million cwt.)^2

Months	Production Forecast Variance	Shrinkage and Loss Estimate Variance	Total Supply Estimate Variance
Sept (Unconditional)	424		424
Oct | Sept	32	0	32
Nov | Oct	8	0	8
Dec | Nov	5	3	8
Jan | Dec	1	3	4
Feb | Jan	0	3	3
Mar | Feb	0	3	3
Apr | Mar	0	3	3
May | Apr	82	3	85
Jun | May	0	0	0
Jul | Jun	28	0	28
Aug | Jul	5	0	5
Final | Aug	79	0	79

resolved uniformly throughout the period of December through May. Thus, we assume the shrinkage and loss estimate changes randomly from month to month with a variance of $\frac{(1.41\%)^2}{6}$ each month during this period. Since the current annual potato production is about 300 million cwt., this gives a monthly variance of 2.98 (million cwt.)^2. In Table 3.4, this figure is entered for the appropriate six months, together with the production information variances discussed in Section 3.3.2.1.

3.3.3 The Information System and Potential Improvements

To quantitatively describe the currently existing supply information system for United States potatoes, we use the variances of the last column of Table 3.4. A reduction in these variances, due to information from any source, results in a benefit that can be calculated with the model of Section 2.2

The condition of perfect information, for example, can be represented by the variances

$$\Sigma^2 = (682,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0).$$

The first component is the sum of the variances appearing in the last column of Table 3.4. It represents the total variability of the supply system. In the case of perfect information, all supply uncertainty is resolved at the start of the crop year, with the publication of a "perfect" forecast. Then the month-to-month changes in the forecast are zero throughout the crop year.

Suppose, as another example, that the information system produces one supply estimate, in September, with standard error 2 percent. Then the residual variance the following August is

$$(.02(300))^2 = 36 \text{ (million cwt.)}^2$$

and the variance of the September forecast itself is

$$682 - 36 = 646 \text{ (million cwt.)}^2 .$$

Thus, the variance vector representing this system is

$$\Sigma^2 = (646,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 36) .$$

These two cases, perfect information and 2 percent error in September will be compared with the current system in Section 3.4.2 giving benefit estimates.

3.4 Results — Potato Distribution

3.4.1 Input Data

Besides the data of Section 3.3.3 describing the information system, our calculations require demand function parameters. These are the elasticity of demand, and typical prices and associated quantities consumed. We use $3.84 per cwt. as the typical price and 300 million cwt. as the typical annual United States consumption. Elasticity estimates for potatoes are scarce, but it is generally agreed that demand is extremely inelastic. For example, in Forecasting Commodity Prices [10], the article on potatoes states,

> "A shortage of a few percent on an annual basis can cause price rises of over 50%. An excess of the same amount can lead to an intolerable glut, generally remediable only by governmental diversion programs." (page 124)

Perhaps the most dependable way of making an elasticity estimate in the context of our present model is to compare the variance of the month-to-month sequence of historical potato prices with the corresponding variance simulated by the model under various elasticity assumptions. For this purpose, Table 3.5 gives average prices received by farmers for potatoes by month from 1960 through 1972, in real (1975) dollars. The variance

Table 3.5 U.S. Average Price Received by Farmers for Potatoes on the 15th of Month, 1975 $/cwt.

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1960	4.46	4.40	5.38	6.28	5.72	4.53	4.92	4.43	3.95	3.54	3.97	3.85
1961	3.66	3.42	3.33	3.57	3.34	3.29	3.31	3.06	2.74	2.42	2.37	2.24
1962	2.30	2.39	2.60	2.69	3.48	4.45	4.25	3.78	3.21	2.86	2.96	2.97
1963	3.07	3.13	3.05	2.87	3.17	3.04	3.73	4.31	3.26	2.71	2.71	2.70
1964	2.92	3.11	2.65	4.30	5.89	7.70	6.76	5.26	4.35	4.54	5.51	7.02
1965	7.77	8.33	8.52	9.39	9.27	9.20	8.87	4.68	3.59	3.46	3.70	3.67
1966	3.84	4.04	4.44	4.86	4.72	3.43	3.26	4.14	3.79	3.50	3.62	4.00
1967	4.01	4.13	3.76	3.28	3.60	3.50	4.60	4.21	3.32	3.03	3.03	2.94
1968	2.94	2.74	2.52	3.29	4.56	4.62	4.98	3.26	3.15	2.99	3.23	3.51
1969	3.60	3.77	4.14	4.29	4.30	4.46	4.02	3.79	3.21	2.90	2.91	3.18
1970	3.50	3.67	4.03	4.14	4.32	4.58	4.10	3.94	3.12	3.01	2.88	2.94
1971	2.92	2.96	3.06	3.09	3.71	3.19	3.36	3.23	2.73	2.57	2.54	2.69
1972	2.67	2.61	2.64	2.59	2.85	3.39	5.24	4.38	3.41	3.01	3.59	3.87

of the sequence from September 1960 through August 1972 is 2.02 ($\$^2$). Our model produces this same variance of the price sequence when run with a demand elasticity of -.26.

It should be noticed that we are not assuming variations in elasticity from month-to-month throughout the year. The -.26 might be called an "equivalent constant elasticity." It is probable that the actual demand elasticities vary significantly throughout the year since processing is seasonal. But the value -.26 serves to calibrate our model well with actual price history. It should also be observed that the historical prices have fluctuated less severely than they would in a simple free market. In times of surplus, the federal government has paid subsidies to those selling potatoes to starch factories. These and temporary programs have prevented prices from going as low as they otherwise would have. Accordingly, the elasticity estimate of -.26 probably leads to a conservative statement of the value of improved information. An alternative demand elasticity estimate of -.06 can be obtained from the remark from Forecasting Commodity Prices quoted above. If "a few percent" is taken to mean 3 percent, the demand elasticity becomes

$$-\frac{3\%}{50\%} = -.06 .$$

We present calculations in the next section based on both assumptions.

3.4.2 Benefit Estimates—Potatoes

Table 3.6 gives the benefits of improved information on potato supply corresponding to two information systems and two demand elasticity estimates. The information system identified as "2% Std. Dev. of Error at Harvest of Fall Crop" is extremely good. It provides only one estimate, in September, covering fall potatoes for the current year and the remaining seasonal groups of the coming calendar year. The remaining uncertainty is resolved at the end of the summer. The current information system, quantitatively described in Table 3.4, has an error with standard deviation about 5 percent in September, decreasing to about 2.9 percent by the end of the crop year the following summer.

The perfect information case puts an absolute ceiling on the possible distribution benefits of improved supply information. In this case it is assumed that the full crop year's actual supply (after shrinkage and loss) is known in September.

Table 3.6 Benefits of Improved Information on U.S. Potato Supply, millions of 1975 $

Elasticity of Demand	2% Std. Dev. of Error* at Harvest of Fall Crop	Perfect Information
-.26	17.6	28.6
-.06	76.3	123.9

*This means error in crop composed of current year fall production and following year production of seasonal groups.

4.0 SUGAR

4.1 Summary of Sugar Market Factors

Sugar (raw centrifugal) is produced from sugar cane and sugar beets. Sugar cane is a perennial grass which grows in tropical and semitropical regions. The sugar beet is a biennial grown in temperate regions. Sugar beet yields are quite sensitive to weather conditions. The uncertainty in European sugar beet yields is probably the primary source of error in world sugar supply estimates.

The United States is one of the few countries growing both sugar cane and sugar beets. It produces about 6 percent of the world's raw centrifugal sugar, but consumes about 11 percent. Thus, it is a major importer. Other large net importers are the U.K., the U.S.S.R., Canada and Japan. Large net exporters include Cuba, Brazil, Australia, Philippines and the Dominican Republic.

Less than half of the sugar moving in international trade prior to 1974 entered the free market. The remainder has been subject to various protected agreements. These included the United States quota system, the agreements of the British Commonwealth, the agreements between France and her former colonies, and the agreements among the various communist countries. Both the United States quota system and the Commonwealth Sugar Agreement ended in 1974. It is probable that in the future, United States imports will be entirely from the free market which will account for most international trade in sugar. Grinding seasons begin in major producing countries in months ranging from June to November. In the United States, the season begins October 1, while in the United Kingdom it begins September 1. In Australia and Brazil, the starting date is June 1, while in Mexico, it is November 1. These dates represent the approximate low stock points in the various regions. By convention in international trade, the worldwide carryover date is taken to be August 31.

4.2 Method of Applying Benefits Model to Sugar Distribution

For worldwide sugar distribution, we use a formulation of the model with three state variables. The first two state variables refer to current supply estimates in the rest of the world (exporting unit) and the United States (importing unit) just as in the model as used for wheat in Volume 1 of this study. The third state variable refers to the production estimate in the United States <u>for the next</u> crop year. In this application, the year is divided into four periods of three months each, and the year is considered to begin in September. That is, as far as the model is concerned, the new crop in both the United States and the rest of the world appears at the beginning of September. Decision making and value functions are based on an infinite horizon. Three decision variables are used—consumption in the rest of the world, consumption in the United States, and the imports to the United States from the rest of the world.

4.2.1 Dynamic Programming Formulation

As in Volume 1 of this study, we use vector notation, with X_t representing the state vector at time t, Y the decision vector, and Φ_t the vector of stochastic terms. The state transformation is

$$x_{(t+1)1} = x_{t1} - y_1 - y_3 + \phi_{t1} ,$$

$$x_{(t+1)2} = x_{t2} - y_2 + y_3 + \phi_{t2} ,$$

$$x_{(t+1)3} = x_{t3} + \phi_{t3} ,$$

for $t \equiv 1, 2, 3 \pmod 4$, and for the final period of a year, in which $t \equiv 0 \pmod 4$,

$$x_{(t+1)1} = x_{t1} - y_1 - y_3 + \pi_2 + \phi_{41} ,$$

$$x_{(t+1)2} = x_{t2} + x_{t3} + y_2 + y_3 + \phi_{42} ,$$

$$x_{(t+1)3} = \pi_1 + \phi_{43} .$$

Notice that the third state variable, the estimated production for the following year, does not change in response to decisions, but only as a result of new information. This applies for the first three periods of the year. In the final period, the potential production tracked by the third state variable is transferred to the second state variable, while the average production term, π_1, becomes the new value in the third state variable. Thus, this variable begins tracking potential production a year farther in the future.

The terms ϕ_{ij} each have mean zero. This is a departure from the notation of volume 1, in which ϕ_{ij} included additional production as well as changes in information. Here, the additional production is represented by π_1 in the importing unit, and π_2 in the exporting unit.

In matrix notation, the state transformation is given by

$$X_{t+1} = X_t + \begin{pmatrix} -1 & 0 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix} Y + \Phi_t$$

for $t \equiv 1, 2, 3 \pmod 4$, and

$$X_{t+1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} X_t + \begin{pmatrix} -1 & 0 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix} Y$$

$$+ \begin{pmatrix} \pi_2 \\ 0 \\ \pi_1 \end{pmatrix} + \Phi_t$$

for $t \equiv 0 \pmod 4$.

Let

$$M = \begin{pmatrix} -1 & 0 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix},$$

$$N_i = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad i = 1, 2, 3,$$

$$N_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\Pi_i = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad i = 1, 2, 3, \quad \Pi_4 = \begin{pmatrix} \pi_2 \\ 0 \\ \pi_1 \end{pmatrix}.$$

Then the state transformation becomes simply

$$X_{t+1} = N_t X_t + MY + \Pi_t + \Phi_t \ .$$

The fundamental value function, W_t, for this situation is exactly as defined in Volume 1. $W_t(X)$ is the maximum, over the possible choices of Y at times $t, t+1, t+2, \ldots,$ of the discounted mean economic value of consumption minus transportation costs in periods $t, t+1, t+2, \ldots$.

The increment accruing during period t to the value function is denoted $w(Y)$. Just as before, it is given by

$$w(Y) = Y^*EY + Y^*F$$

where

$$E = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \gamma & 0 \\ 0 & 0 & \tau_1 \end{pmatrix}, \quad F = \begin{pmatrix} \beta \\ \delta \\ -\tau \end{pmatrix},$$

and the demand functions and transportation costs are

$$\text{price} = 2\alpha y_1 + \beta \text{ (rest of world)},$$
$$\text{price} = 2\gamma y_2 + \delta \text{ (United States)},$$
$$\text{cost} = \tau y_3 + \tau_1 y_3^2 .$$

We denote by u the incremental value function for the United States alone. It is given by

$$u(Y) = \gamma y_2^2 + \delta y_2 - (2\alpha y_1 + \beta) y_3$$
$$- \tau y_3 - \tau_1 y_3^2 .$$

We have assumed here that the importer pays the price prevailing in the exporting unit as well as the transportation costs. Letting

$$C = \begin{pmatrix} 0 & 0 & -\alpha_1 \\ 0 & \gamma & 0 \\ -\alpha_1 & 0 & -\tau_1 \end{pmatrix} , \quad D = \begin{pmatrix} 0 \\ \delta \\ \beta - \tau \end{pmatrix} ,$$

the United States incremental value function can be written

$$u(Y) = Y^*CY + Y^*D .$$

The principle of optimality for this case is exactly as given in Volume 1 of this study (Equation 4, Section 3.4), and the solution procedure is the same with the following exception. In Volume 1, the exposition concerned a two-dimensional state space and the value function approximation was based on a grid of twenty-five points. In the present case, the state space is three dimensional, and we use a grid of twenty-seven points, based on three representative values of each state variable. Consult the appendix for a precise statement of the solution procedure.

4.2.2 Backward Induction Step

Given the coefficients Q_{t+1}, P_{t+1}, and r_{t+1} of the value function W_{t+1} at time $t+1$, the coefficients Q_t, P_t, and r_t of the approximation to W_t are found by evaluating W_t on a grid of 27 points in the state space, and then performing a least squares fit of a quadratic form to the evaluated function. Each evaluation requires the solution of a small quadratic programming problem. If X is the grid point at hand, the maximization is as follows.

$$W_t(X) = \max_Y \left\{Y^*RY + Y^*S + T\right\}$$

subject to $y_1 \geq 0$, $y_3 \geq 0$, $x_2 \geq y_2 \geq 0$, $x_1 \geq y_1 + y_3$, where

$$R = E + \rho M^* Q_{t+1} M ,$$

$$S = F + \rho M^*(P_{t+1} + 2Q_{t+1}\ (N_tX + \Pi_t)),$$

$$T = \rho[(N_tX)^*Q_{t+1}\ (N_tX) + 2(N_tX)^*Q_{t+1}\Pi_t$$

$$+ \Pi_t{}^*Q_{t+1}\Pi_t + \overline{\Phi_t{}^*Q_{t+1}\Phi_t}$$

$$+ (N_tX + \Pi_t)^*P_{t+1} + r_{t+1}].$$

Assuming the covariances are zero, the term $\overline{\Phi_t{}^*Q_{t+1}\Phi_t}$ can be expanded to $q_{11}\sigma_{t1}{}^2 + q_{22}\sigma_{t2}{}^2 + q_{33}\sigma_{t3}{}^2$ where $\sigma_{ij}{}^2$ is the variance of ϕ_{ij} and q_{ij} are the elements of Q_{t+1}. The variances are obtained from the forecast variances $(\sigma_i{}^2)$ of Section 2.1 as follows. In the exporting unit (rest of the world), we have information only during the current year, m=4, and we set

$$\sigma_{11}{}^2 = \sigma_1{}^2\ ,$$

$$\sigma_{21}{}^2 = \sigma_2{}^2\ ,$$

$$\sigma_{31}{}^2 = \sigma_3{}^2\ ,$$

$$\sigma_{41}{}^2 = \sigma_0{}^2\ .$$

In the importing unit, we have information during the current year and the coming year. Information on the coming year, tracked in state variable x_3, is represented by the components $\sigma_0{}^2$, $\sigma_1{}^2$, $\sigma_2{}^2$, $\sigma_3{}^2$ of the forecast variance vector $\Sigma^2_{ROW} = (\sigma_0{}^2, \sigma_1{}^2, \ldots, \sigma_7{}^2)$. Accordingly, we have

$$\sigma_{13}{}^2 = \sigma_1{}^2 ,$$

$$\sigma_{23}{}^2 = \sigma_2{}^2 ,$$

$$\sigma_{33}{}^2 = \sigma_3{}^2 ,$$

$$\sigma_{43}{}^2 = \sigma_0{}^2 .$$

Information on the current year, tracked in state variable x_1, is represented by components $\sigma_4{}^2, \sigma_5{}^2, \sigma_6{}^2, \sigma_7{}^2$, of Σ^2_{ROW}. Accordingly,

$$\sigma_{12}{}^2 = \sigma_5{}^2 ,$$

$$\sigma_{22}{}^2 = \sigma_6{}^2 ,$$

$$\sigma_{32}{}^2 = \sigma_7{}^2 ,$$

$$\sigma_{42}{}^2 = \sigma_4{}^2 .$$

4.3 Current Information System—Sugar Crops

To estimate the level of uncertainty concerning worldwide production of sugar, we consider data from two primary sources. The Crop Reporting Board of the USDA published estimates of sugar crop production in the United States during the growing season, as well as a final estimate after the end of the crop year. The Foreign Agricultural Service of the USDA publishes final estimates for each crop year of sugar production in various countries and the world as a whole. During the growing season, the best source of information on the rest of the world's sugar production is F.O. Licht's International Sugar Report, published monthly in Ratzeburg, West Germany.

To estimate the variability of the sugar production system itself, independent of information, we consider the trends in the final production estimates and the average deviations from these trends. Table 4.1 gives this information, with the results that the standard deviation of production uncertainty appears to be 9.15 percent in the United States and 3.31 percent in the rest of the world. Using production levels of 1975-6 for normalization, the variances are

$$(.0915 \times 5.188)^2 = 0.225 \text{ (million tons)}^2$$

for the United States production and

Table 4.1 United States and Rest of World Production of Sugar (raw value)

Crop Year	Production, millions of metric tons		Extrapolated Forecast, millions of metric tons		% Error of Extrapolated Forecast	
	United States	Rest of World	United States	Rest of World	United States	Rest of World
1965-6	3.559	59.290	—	—	—	—
1966-7	3.696	60.803	3.56	59.29	-3.71	-2.49
1967-8	3.765	62.452	3.83	62.32	1.81	-0.22
1968-9	4.285	63.320	3.88	64.01	-9.47	1.09
1969-0	4.120	67.760	4.39	64.90	6.50	-4.22
1970-1	4.149	66.360	4.40	68.56	6.01	3.32
1971-2	4.279	66.328	4.40	69.04	3.63	4.09
1972-3	4.792	70.506	4.45	69.12	-7.13	-1.96
1973-4	4.189	76.285	4.74	71.15	13.16	-6.73
1974-5	4.017	76.823	4.64	74.89	15.57	-2.52
1975-6	5.188	76.539	4.50	77.46	-13.20	1.21
				Standard Deviation	9.15	3.31

$$(.0331 \times 76.539)^2 = 6.418 \text{ (m-llion tons)}^2$$

for the rest of the world production.

The figures for the United States seem reasonable, but the statement that the sugar production in the rest of the world can be predicted to within 3.31 percent (standard error) by trend extrapolation alone seems less convincing. The general opinion of the trading community, the text of F.O. Licht's International Sugar Report, and the historical price fluctuations in the sugar market, all suggest that the actual uncertainty is substantially higher. Thus, it is likely that the final estimates themselves are inaccurate.

Estimates of sugar cane and sugar beet production in the United States are published by the USDA Crop Reporting Board each month from August through December. Through 1971, a July estimate of each crop was also published. For our purposes, any estimate published before September is treated as if available in the fourth period of the crop year (June-August). The September estimate is used to represent the fifth period (first period of the new crop year), and the December estimate is used to represent the sixth period. These assumptions lead to the "forecast" matrix given in Table 4.2.

Processing these "forecasts" as described in

Table 4.2 United States Sugar Production Forecasts Adapted from Crop Reporting Board Estimates on Sugar Beets and Sugar Cane.				
	Sugar Production (raw value), millions of metric tons			
Crop Year	Period 4 (June)	Period 5 (Sept)	Period 6 (Dec)	Final (June)
1965-6	3.25	3.30	3.18	3.56
1966-7	3.68	3.82	3.72	3.70
1967-8	3.48	3.55	3.63	3.77
1968-9	4.23	4.32	4.32	4.29
1969-0	4.37	4.44	4.51	4.12
1970-1	4.22	4.32	4.35	4.15
1971-2	4.31	4.38	4.39	4.28
1972-3	4.58	4.64	4.87	4.79
1973-4	4.40	4.41	4.43	4.19
1974-5	4.02	4.05	3.95	4.02

Chapter 2, and normalizing to 1975-6 production levels, we obtain the variance vector

$$\Sigma_{US}^2 = (0,\ 0,\ 0,\ .170,\ .001,\ .018,\ 0,\ .085).$$

Estimates of world sugar production are more sporadic, although some relevant information comes very early in the year. European sugar beet acreage estimates are discussed by F.O. Licht's as early as February. Analysis of time series taken from F.O. Licht's and elsewhere has not been conclusive with respect to the variance sequence needed for our benefits calculation. For the case of sugar, however, there is a useful indication of the current quality of information in the form of the historical price series. Since sugar has no substitutes, it is safe to assume that for long periods of time (free of political complications such as wars), sugar price fluctuations reflect supply conditions and supply information only. A rough indication of information quality can thus be obtained from the model itself by adjusting the input variances so that a simulation of the market produces the same price variability as is observed in historical data. For this purpose, we vary the total variability of the rest of the world's sugar production, while assuming that the uncertainty is resolved during the year according to a simple pattern. Namely, we assume that of a total variance T , ½T is resolved with the September estimate, while the remaining ½T is resolved linearly throughout the marketing year. Thus, the variance vector is

$$\Sigma^2_{ROW} = (\tfrac{1}{2}T, \tfrac{1}{6}T, \tfrac{1}{6}T, \tfrac{1}{6}T) .$$

The period chosen for tracking historical price variability is 1966 through 1972. Table 4.3 gives quarterly average prices (in January, 1975 \$/metric ton) of world sugar for this interval. The residual variance after detrending of this entire series is 1011 $(\$/\text{ton})^2$. Simulation of the system produces this variance when

$$T = 21.5$$

and $\Sigma^2_{US} = (0, 0, .170, .061, .018, 0, 0, .085)$.

4.4 Results—Sugar Distribution

4.4.1 Input Data

The demand elasticities, transportation costs, and production averages used for our calculations are given in Table 4.4.

Table 4.3 World Sugar Prices in Constant (January 1975) Dollars, 1966-1972.

Year	Price per metric ton, Quarterly Average			
	1	2	3	4
1966	78.99	67.64	57.55	51.24
1967	53.54	82.66	61.68	76.12
1968	72.22	64.20	55.26	80.94
1969	113.26	131.72	114.64	103.86
1970	112.23	128.28	133.90	141.35
1971	163.59	150.18	143.99	164.28
1972	293.48	239.37	216.67	272.27

Table 4.4 Sugar Data other than on Information Systems					
	Demand Parameters			Transportation Cost to United States, $/ton	Mean Annual Production, millions of tons
	Elasticity	Mean Price, $/ton	Mean Consumption, millions of tons/year		
United States	-.25	331	9.0	0	5.2
Rest of World	-.35	311	72.7	10	76.5

The variance vectors describing the information systems are given in Table 4.5.

"Case 1" differs from the current system in providing improved information on the United States. The forecast accuracies are assumed the same as in the current system, but they come one period earlier (3 months).

$$\Sigma^2_{US} = (0,\ 0,\ .170,\ .001,\ .018,\ 0,\ 0,\ .085)\ .$$

"Case 2" comprises the improvements just described on the United States and an improvement on the rest of the world. For this case the September forecast is assumed to have a standard error of three percent, and no further estimate is made. The market is assumed to "discover" the truth at the start of the final period (June). Thus, the residual variance is

$$(.03 \times 76.54)^2 = 5.27 \text{ (million tons)}^2 .$$

and the variance of the September forecast is

$$21.5 - 5.27 = 16.23 \text{ (million tons)}^2.$$

4.4.2 Distribution Benefits—Sugar

The benefits model produces value functions

Table 4.5 Forecast Variance Vectors Describing Information Systems—Sugar, (millions of tons)^2

	Current System	Improvements Case 1	Improvements Case 2
United States	0	0	0
	0	0	0
	0	.170	.170
	.170	.001	.001
	.001	.018	.018
	.018	0	0
	0	0	0
	.085	.085	.085
Rest of World	10.75	10.75	16.23
	3.58	3.58	0
	3.58	3.58	0
	3.58	3.58	5.27

for the United States and the entire world whose coefficients are given in Table 4.6. When these are applied to the various information systems described in Table 4.5, benefits are determined as given in Table 4.7.

It is significant that improvements in knowledge of the exporter's production are beneficial to the exporter, but not to the importer. The "domestic" category of benefits shows an improvement in this case, but it is more than compensated for by the disbenefit in the trade category. Here and throughout this study, the "domestic" or "within" category refers to changes in the integral under the demand function, while the "trade" category refers to revenues and expenditures for exports.

Table 4.5 Value Coefficients Relative to Zero Variances—Sugar

Value to	Forecast Variance on	Coefficients* of Value Function, (\$ million/year)/(millions of tons)^2											
		United States								Rest of World			
R.O.W.	Within	-12.63	-12.86	-13.15	-13.80	-12.36	-16.55	-13.81	31.93	-4.44	-5.70	-7.91	-12.63
	Trade	14.36	14.66	15.04	15.87	13.97	17.17	11.43	-42.86	-0.53	-0.65	-0.79	-1.13
	Total	1.73	1.80	1.90	2.07	1.61	0.63	-2.37	-10.94	-4.97	-6.35	-8.70	-13.76
U.S.	Domestic	-6.56	-6.67	-6.78	-6.83	-15.74	-23.60	-60.58	-198.69	-0.13	-0.12	-0.01	0.27
	Trade	-14.54	-14.84	-15.23	-16.07	-14.17	-17.29	-11.26	44.22	0.53	0.65	0.79	1.11
	Total	-21.10	-21.51	-22.01	-22.90	-29.91	-40.89	-71.84	-154.47	0.40	0.53	0.78	1.38
World	Total	-19.37	-19.71	-20.12	-20.83	-28.29	-40.26	-74.22	-165.41	-4.57	-5.82	-7.93	-12.38

*Value obtained by scalar product of coefficient vector with forecast variance vector.

Table 4.7 Distribution Benefits—Sugar, millions of 1975 $ per year

	United States			Rest of the World		
	Domestic	Trade	Total	Within	Trade	Total
Case 1	0.16	0.20	0.36	0.19	-0.20	-0.01
Case 2	0.36	-0.15	0.21	3.31	0.15	3.46

5.0 SOYBEANS

5.1 Summary of Soybeans Market Factors

The United States is the largest grower of soybeans in the world, and supplies near 85 percent of the export market. About 40 percent of United States production is exported. Brazil is the second largest exporter, supplying about 12 percent of the export market. The world's largest importer is Japan. Other major importers are West Germany, Spain, The Netherlands, Italy and Denmark.

Almost all soybeans are converted into products—soybean oil and soybean meal. In the United States, most soybean oil is used for food products, such as cooking and salad oils, shortening and margerine. A small amount is used for industrial purposes. Almost all soybean meal is used for high protein animal feed.

Soybeans prices are closely related to the product prices. The price of soybean meal has typically accounted for about 63 percent of combined product value and oil 37 percent.

The soybeans crop or marketing year runs from September 1 through the following August 31. The United States peak harvest month is October. In Brazil, the harvest months are April and May, while in China, the third largest producer (but now a net importer), the harvest months are August through November.

5.2 Method of Applying Benefits Model to Soybeans Distribution

Just as in the case of sugar distribution, we use here a formulation of the model with three state variables. The first and third state variables refer to the exporting unit, which in the case of soybeans is the United States. The second state variable refers to the importing unit, which is the rest of the world. The crop year is divided into quarters and is considered to begin September 1, which is the conventional starting date in the United States.

Thus, the equations and methods are exactly as in the case of sugar distribution, with the exception that the United States is the exporting unit instead of the importing unit.

5.3 Current Information System—Soybeans

United States

Since the United States is by far the world's largest soybeans producer, information on the United States crop is the most significant. The Crop Reporting Board of the USDA publishes acreage estimates for the coming (September) crop in mid-March, and then another in July. Market agents can construct an early forecast of production by combining these figures with average abandonment and yield adjusted for trend. Then in August, September, October, November, and December, the official production estimates are published. To represent current information on United States soybeans production for our benefits model, we need representative "forecasts" at quarterly intervals. During the crop year, beginning in September, we use the published September and December estimates, for our period 1 and period 2 "forecasts." The published "final" estimate is assumed to be known in the market at the start of period 4. In advance of the crop year, the first information specific to the coming crop is assumed to come with the March acreage estimate, at the beginning of period 3. This is converted into a production forecast by linear extrapolation of the least squares fit of previous values of the ratio of actual production to projected acreage in March. The July acreage estimate is used the same way to produce a period 4 "forecast." Table 5.1 gives the March and July acreage figures from 1960 through 1974, together with the final production figures and the constructed "forecasts." The complete "forecast" matrix describing the current information system is given in Table 5.2

As described in Section 2.1, this matrix leads to a vector of variances $(\sigma_0^2, \sigma_1^2, \ldots, \sigma_7^2)$,

Table 5.1 Construction of March "Forecast" for U.S. Soybeans Production

Crop Year	March Acreage Estimate, millions of acres		Final Production Report, millions of tons	Constructed March "Forecasts," millions of tons	
	March	July		March	July
1960-1	24.7	23.6	15.1	15.1	15.1
1961-2	26.4	27.1	18.5	17.8	17.3
1962-3	23.8	27.9	18.2	18.9	20.2
1963-4	29.9	29.1	19.0	19.5	19.5
1964-5	31.8	30.9	19.1	20.6	20.4
1965-6	34.3	34.7	23.0	21.2	21.8
1966-7	37.1	36.9	25.3	24.1	23.8
1967-8	40.6	40.1	26.6	27.4	26.7
1968-9	41.8	40.9	30.1	28.2	27.3
1969-0	43.0	41.7	30.8	30.2	29.2
1970-1	43.1	41.6	30.7	30.9	30.0
1971-2	46.5	42.8	32.0	33.8	31.5
1972-3	45.5	45.6	34.6	33.0	34.2
1973-4	48.8	55.7	42.6	36.4	42.5
1974-5	54.9	52.5	33.9	43.5	40.6

Crop Year	F_0 (Extrapolated)	F_3 (March)	F_4 (June)	F_5 (September)	F_6 (December)	F_7 (March)	F_8 (June)
1960-1	15.1	15.1	15.1	15.4	15.2	15.2	15.1
1961-2	15.1	17.8	17.3	19.6	18.9	18.9	18.5
1962-3	21.9	18.9	20.2	18.2	18.4	18.4	18.2
1963-4	20.4	19.5	19.5	19.8	19.1	19.1	19.0
1964-5	20.6	20.6	20.4	20.4	19.0	19.0	19.1
1965-6	20.5	21.2	21.8	23.6	23.0	23.0	23.0
1966-7	23.0	24.1	23.8	25.2	25.4	25.4	25.3
1967-8	25.5	27.4	26.7	27.3	26.5	26.5	26.6
1968-9	27.5	28.2	27.3	29.4	29.4	29.4	30.1
1969-0	30.2	30.2	29.2	28.7	30.4	30.4	30.8
1970-1	32.2	30.9	30.0	30.8	30.9	30.9	30.7
1971-2	33.4	33.8	31.5	32.3	31.8	31.8	32.0
1972-3	34.6	33.0	34.2	35.0	34.7	34.7	34.6
1973-4	36.2	36.4	42.5	43.5	42.6	42.6	42.6
1974-5	39.7	43.5	40.6	35.8	33.6	33.6	33.9

Table 5.2 "Forecast" Matrix for U.S. Soybeans Production, millions of metric tons

representing the performance of the current information system. σ_0^2 represents the unconditional variance of the forecast (F_1) a full year in advance (September). σ_1^2 represents the variance of the December forecast (F_2) of the previous year, conditional on F_1, etc. The variance vector calculated from Table 5.2 is

$$\Sigma^2_{U.S.} = (0,\ 0,\ 8.09,\ 6.58,\ 6.66,\ 1.24,\ 0,\ 0.13)\ .$$

Rest of the World

Information on soybeans production on the rest of the world is not extensive. The best source of public data is the Foreign Agricultural Service of the USDA, which publishes the monthly report, World Agricultural Production and Trade. Usually, either the October or the December issue gives an estimate of world soybeans production for the current crop year. Table 5.3 gives a sample of recent years' soybeans production forecasts for the rest of the world, together with "final" value and percent error.

Estimating that the standard deviation of the rest of the world's soybeans production in advance of any crop-specific information is 15 percent, and using the 1975 production of 21.2 million tons as the normalizing

Table 5.3 Rest of the World Soybean Production Forecasts and Errors

Crop Year	Production, millions of metric tons October	December	Final	% Error
1960-1	11.5	11.5	10.6	8.49
1961-2	10.6	—	10.0	6.00
1962-3	11.6	—	9.9	17.17
1963-4	10.1	—	9.3	8.60
1966-7	10.2	—	9.5	7.36
1967-8	9.8	—	9.5	3.16
1970-1	9.9	—	11.1	10.81
1971-2	11.7	—	11.6	8.67
1972-3	13.1	—	12.9	1.55
1973-4	15.0	—	15.3	1.96
1974-5	—	17.6	18.4	4.35

Standard deviation of error = 8.71%

value, we obtain 10.11 (million tons)^2 as the total crop variance. From Table 5.3, the residual variance after the fall estimate is

$$(.0871 \times 21.2 \text{ million tons})^2$$

or 3.41 (million tons)^2. Thus, the unconditional variance of the first forecast itself is

$$10.11 - 3.41 = 6.70 \text{ (million tons)}^2 .$$

For our quarterly benefits model, we treat the forecast as coming in the first period (September). The variance vector describing information system performance is thus

$$\Sigma^2_{ROW} = (6.70, 0, 0, 3.41, 0).$$

5.4 Results—Soybeans Distribution

5.4.1 Input Data

The demand function and transportation cost parameters used for our calculations are given in Table 5.4.

The variance vectors used to describe the information systems are given in Table 5.5. Three possible

Table 5.4 Soybeans Data other than on Information Systems

	Demand Parameters			Transportation Cost From United States, $/ton	Mean Annual Production, mill. metric tons
	Elasticity	Mean Price, $/ton	Mean Consumption mill. tons/year		
United States	-.4	220	24.1	0	40.1
Rest of the World	-.4	230	37.2	10	21.2

Table 5.5 Variance Vectors Describing Information Systems—Soybeans, (millions of tons)^2

	United States								Rest of the World			
Current System	0	0	8.09	6.58	6.66	1.24	0	.13	6.70	0	0	3.41
Improvement Case 1	0	0	8.09	6.58	6.66	1.24	0	.13	8.50	0	0	1.61
Improvement Case 2	0	8.09	6.58	6.66	1.24	0	0	.13	6.70	0	0	3.41
Improvement Case 3	0	8.09	6.58	6.66	1.24	0	0	.13	9.71	0	0	.40

improved systems are compared with the current system. "Improvement Case 1" provides information improvement on the production on the rest of the world, but not in the United States. This system gives greater accuracy in the September estimate, the standard error reduced from 8.7 percent to 6 percent. "Improvement Case 2" provides improvement only on the United States, by providing the same month-by-month accuracies as the current system, but at times one period earlier (three months). "Improvement Case 3" provides advantages over the current system both in the United States and on the rest of the world. The accuracy in estimating the United States production is the same as in "Improvement Case 2," but the September estimate in the rest of the world has standard error only 3 percent.

5.4.2 Distribution Benefits—Soybeans

The benefits associated with each of these cases are presented in Table 5.6. The benefits are separated, both for the United States and for the rest of the world, into "domestic" and "trade" categories.

As in the case of sugar, information on the exporter's production (in this case the United States) benefits the exporter, but not the importer. The exporter, however, benefits from improved information on both regions.

Table 5.6 Distribution Benefits—Soybeans, millions of 1975 dollars per year						
	United States			Rest of the World		
	Domestic	Trade	Total	Domestic	Trade	Total
Case 1	+.198	.36	.54	2.106	-.36	1.73
Case 2	5.92	3.67	9.57	.82	-3.64	-2.76
Case 3	6.25	4.27	10.47	4.34	-4.24	.12

6.0 CORN

6.1 Summary of Corn Market Factors

The United States produces over 40 percent of the world's corn supply, and it exports nearly 20 percent of its own production. Domestically, corn is used primarily for livestock feed, but about 10 percent of domestic consumption is for food, industry and seed. The rapidly increasing export demand is also primarily for corn as a feed grain.

There has been a long term upward trend in United States corn production since the 1930's, brought about solely by yield increases. Acreage in recent years has been fairly stable.

In the northern hemisphere, the corn harvest is concentrated mainly in September and October. In South America, harvest is in March and April. The crop year in the United States is conventionally taken as beginning October 1.

Besides the United States, major producers of corn are the U.S.S.R., China, Brazil, France and Argentina. Exports from the United States go primarily to Japan and western Europe.

6.2 Method of Applying Benefits Model to Corn Distribution

For corn, we use three state variables, with two refering to the United States, the third to the rest of the world. The crop year is divided into intervals 2 months in length, and is considered to begin on October 1. This applies to both the United Stated (exporting unit) and the rest of the world (importing unit).

6.3 Current Information System—Corn

6.3.1 United States

The Crop Reporting Board of the USDA publishes production forecasts for corn well in advance of the marketing year. The first forecast comes in July, while the crop year begins on October 1.

Table 6.1 gives the monthly production estimates, together with final estimates, for the crop years 1960-1 through 1974-5. For the purposes of our 6-period benefits model, we form the "forecast" matrix shown in Table 6.2 by selecting the appropriate forecasts for each period, and using extrapolation of previous final estimates for the "forecasts" in advance of July (F_5).

The forecast variance vector (described in Chapter 2) for the case of United States corn is calculated from the matrix of Table 6.2, and is

$$\Sigma^2_{U.S.} = (0,\ 0,\ 0,\ 0,\ 132.37,\ 66.69,\ 35.61,\ 10.24,\ 0,\ 0,\ 0,\ 7.93)$$

Table 6.1 United States Corn Production Estimates Published by USDA Crop Reporting Board, millions of metric tons

Crop Year	Jul	Aug	Sep	Oct	Nov	Dec	Final
1960-1	—	—	—	—	—	—	99.2
1961-2	80.65	85.15	80.26	89.60	90.14	92.06	91.4
1962-3	89.36	90.16	88.52	89.20	91.20	92.55	91.6
1963-4	97.77	98.09	100.05	101.84	102.44	103.67	102.1
1964-5	98.77	98.69	92.47	90.54	89.95	90.14	88.5
1965-6	99.37	104.04	105.25	106.16	104.86	105.95	104.2
1966-7	107.23	101.13	103.89	104.07	104.90	104.23	105.9
1967-8	114.52	118.16	119.23	119.83	119.28	119.95	123.5
1968-9	113.09	115.72	117.77	116.52	112.78	111.13	113.0
1969-0	108.88	109.47	109.55	110.51	112.89	116.28	119.0
1970-1	122.43	119.20	111.84	106.39	104.25	104.39	105.5
1971-2	110.50	135.77	133.75	137.16	141.02	140.73	143.3
1972-3	128.07	125.68	130.17	133.76	137.18	139.04	141.6
1973-4	149.35	143.81	146.52	146.39	144.23	143.35	143.3
1974-5	140.90	126.14	126.87	119.83	117.39	118.15	126.9

Table 6.2 "Forecast" Matrix for United States Corn Production, millions of metric tons

Crop Year	F_0 (Extrapolated)	F_1 (Oct)	F_2 (Dec)	F_3 (Feb)	F_4 (Apr)	F_5 (Jun)	F_6 (Aug)	F_7 (Oct)	F_8 (Dec)	F_9 (Feb)	F_{10} (Apr)	F_{11} (Jun)	F_{12} Final
1960-1	99.24	99.24	99.24	99.24	99.24	99.24	99.24	99.24	99.24	99.24	99.24	99.24	99.24
1961-2	99.24	99.24	99.24	99.24	99.24	80.65	85.14	89.60	92.06	92.06	92.06	92.06	91.39
1962-3	83.54	83.54	83.54	83.54	83.54	89.36	90.16	89.19	92.55	92.55	92.55	92.55	91.59
1963-4	86.43	86.43	86.43	86.43	86.43	97.77	98.09	101.83	103.67	103.67	103.67	103.67	102.08
1964-5	98.26	98.26	98.26	98.26	98.26	98.77	98.69	90.53	90.13	90.13	90.13	90.13	88.49
1965-6	91.32	91.32	91.32	91.32	91.32	99.37	104.04	106.15	105.95	105.95	105.95	105.95	104.20
1966-7	98.83	98.83	98.83	98.83	98.83	107.22	101.12	104.06	104.22	104.22	104.22	104.22	105.90
1967-8	103.63	103.63	103.63	103.63	103.63	114.52	118.15	119.82	119.94	119.94	119.94	119.94	123.50
1968-9	115.08	115.08	115.08	115.08	115.08	113.08	115.72	116.51	111.12	111.12	111.12	111.12	113.00
1969-0	117.33	117.33	117.33	117.33	117.33	108.87	109.47	110.50	116.28	116.28	116.28	116.28	119.00
1970-1	121.03	121.03	121.03	121.03	121.03	122.43	119.20	106.38	104.39	104.39	104.39	104.39	105.50
1971-2	118.51	118.51	118.51	118.51	118.51	118.51	135.76	137.15	140.72	140.72	140.72	140.72	143.30
1972-3	129.19	129.19	129.19	129.19	129.19	128.07	125.68	133.75	139.03	139.03	139.03	139.03	141.60
1973-4	136.38	136.38	136.38	136.38	136.38	149.34	143.80	146.38	143.34	143.34	143.34	143.34	143.30
1974-5	142.14	142.14	142.14	142.14	142.14	142.14	126.14	119.83	118.14	118.14	118.14	118.14	126.90

6.3.2 Rest of the World

The monthly Grain Bulletin, published by the Commonwealth Secretariat in London, gives corn production estimates regularly for several of the world's major producers. Though relatively few countries are included, and there are numerous gaps in the data for those that are, these figures provide some help in estimating the current state of public information on corn production outside the United States. The estimates in Table 6.3 were prepared by ECON from the published estimates of Grain Bulletin, by extrapolation to the world scale of production estimates for Argentina, South Africa, Mexico, and Yugoslavia. The final estimates, however, are taken from the USDA Foreign Agricultural Service reports. By adjoining F_0, the extrapolated trend forecast, and selecting estimates corresponding to the starts of the six periods, we obtain the "forecast" matrix given in Table 6.4. Included in this table are the "error" variances and standard deviations for each period's forecast, normalized to the scale of the 1974-5 crop. These figures show some peculiarities. The extrapolated "forecast" appears markedly superior to those based on publications during the year! And late in the marketing year, the June estimate appears less accurate than the April estimate. Further, the extrapolated "forecast" is amazingly good (standard error 6%), and the only estimates without substantial bias are the extrapolated and the October forecasts.

Table 6.3 Corn Production Estimates, Projected to World Scale (excluding United States) from Grain Bulletin Figures for Argentina, South Africa, Mexico, and Yugoslavia, millions of metric tons

Crop Year	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun	Final
1960-1	-	-	-	118.39	-	121.71	102.85	103.90	118.39	92.88
1961-2	105.57	-	-	89.78	-	109.93	99.96	103.13	99.84	96.13
1962-3	104.04	-	-	-	102.79	-	102.51	100.60	-	98.1
1963-4	97.60	-	-	103.23	-	100.49	100.55	103.07	103.41	102.6
1964-5	98.72	-	-	-	-	-	110.17	108.69	109.73	110.5
1965-6	71.53	-	120.47	121.50	-	131.42	121.50	135.38	128.52	114.7
1966-7	132.70	153.02	163.18	-	158.92	153.96	166.82	163.29	165.95	133.5
1967-8	-	-	143.90	-	138.56	-	132.91	130.69	133.25	130.0
1968-9	132.70	130.83	132.70	136.39	132.70	106.94	132.26	132.12	-	130.3
1969-0	-	143.46	146.28	-	-	167.45	160.13	159.13	140.90	140.9
1970-1	163.65	-	-	-	153.63	169.40	166.94	168.27	-	149.6
1971-2	178.60	166.72	-	149.73	152.65	143.76	156.04	159.98	159.19	148.9
1972-3	163.18	193.57	163.18	162.14	157.60	166.24	148.84	148.15	153.82	144.7
1973-4	163.18	165.66	-	179.03	199.74	200.92	202.89	193.52	192.02	188.3
1974-5	170.85	152.75	167.95	194.99	181.19	187.50	186.46	183.16	164.36	170.6

Table 6.4 "Forecast" Matrix for Rest of the World Corn Production, millions of metric tons

Crop Year	F_o (Extrapolated)	F_1 (Oct.)	F_2 (Dec.)	F_3 (Feb.)	F_4 (Apr.)	F_5 (Jun.)	F_6 (Final)
1960-1	92.9	92.9	92.9	118.4	102.9	118.4	92.9
1961-2	92.9	105.6	105.6	89.8	100.0	99.8	96.1
1962-3	99.4	104.0	104.0	102.8	102.5	100.6	98.1
1963-4	100.9	97.6	97.6	103.2	100.6	103.4	102.6
1964-5	105.2	98.7	98.7	98.7	110.2	109.7	110.5
1965-6	112.6	71.5	120.5	121.5	121.5	128.5	114.7
1966-7	118.2	132.7	163.2	158.7	166.8	166.0	133.5
1967-8	131.4	131.4	143.9	138.6	132.9	133.3	130.0
1968-9	136.8	132.7	132.7	132.7	132.3	132.1	130.3
1969-0	139.9	139.9	146.3	146.3	160.1	140.9	140.9
1970-1	145.9	163.7	163.7	153.6	166.9	168.3	149.6
1971-2	152.9	178.6	166.7	152.7	156.0	159.2	148.9
1972-3	157.3	163.2	163.2	157.6	148.8	153.8	144.7
1973-4	159.1	163.2	165.7	199.7	202.9	192.0	188.3
1974-5	172.7	170.9	168.0	181.2	186.5	164.4	170.6
Mean "Error"	[illegible]	-0.8	7.3	9.0	11.5	11.2	---
Mean Squared "Error"	103.0	488.7	284.5	297.5	246.2	363.1	---
R.M.S. "Error"	10.1	22.1	16.9	17.2	15.7	19.1	---

It seems evident from these statistics that the final estimates, while providing a rather smooth series which is easy to forecast, are probably not themselves accurate enough to serve as a basis for our statistical analysis.

The variances of month-to-month forecast differences can be calculated, however, without consideration of the final estimates. This calculation gives us all but the first and last elements of the forecast variance vector Σ^2_{ROW}. From the data of Table 6.4, we obtain

$$\Sigma^2_{ROW} = (\sigma^2_0, 1109.4, 330.15, 162.04, 149.8, \sigma^2_5).$$

If the final estimates were taken as truth, the final variance would be 228.14, but if instead we assume that information continues to improve smoothly through the marketing year, the residual variance should not greatly exceed σ^2_4, which is 149.8. Let us assume for our estimates that $\sigma^2_5 = \sigma^2_4$. We will make no assumption about σ^2_0, since its value is not needed for benefit analysis (we will not be able to calculate the actual loss function, but we will be able to calculate its change in going from the current system to a LANDSAT system). Denoting by T the sum of the elements of the forecast variance vector, we have

$$\Sigma^2_{ROW} = (T - 1701.2, 1109.4, 330.2, 162.0, 149.8, 149.8)$$

6.4 Results—Corn Distribution

6.4.1 Input Data

The demand parameters, transportation costs, and production averages used for the calculations of this section are given in Table 6.5.

The variance vectors used to describe the information systems are given in Table 6.6. Three possible improved systems are compared with the current system.

The first, improvement case 1, does not change the quality of information on the United States, and provides an estimate with standard error 6 percent on the rest of the world's production at the start of the crop year, October 1. With this system, there is no further estimate, but the true production is known the following August 1.

Thus, the standard error of the October 1 estimate is

$$.06 \times 170.6 = 10.236 \text{ million tons}$$

The variance of the residual uncertainty after this forecast is

$$(10.236)^2 = 104.78 \text{ (million tons)}^2,$$

Table 6.5 Corn Data other than on Information Systems

	Demand Parameters			Transportation Cost From United States, $/ton	Mean Annual Production, millons of tons
	Elasticity	Mean Price, $/ton	Mean Consumption, millons of tons/year		
United States	-.36	124	115	0	145
Rest of the World	-.36	134	215	10	185

Table 6.6 Forecast Variance Vectors Describing Information Systems—Corn, (millions of metric tons)²

	Current System	Improvements		
		Case 1	Case 2	Case 3
United States	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	132.4	132.4
	132.4	132.4	66.7	66.7
	66.7	66.7	35.61	35.61
	35.61	35.61	10.24	10.24
	10.24	10.24	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	7.93	7.93	7.93	7.93
Rest of World	T-1901.2	T-104.8	T-1901.2	T-26.2
	1109.4	0	1109.4	0
	330.2	0	330.2	0
	162.0	0	162.0	0
	149.8	0	149.8	0
	149.8	104.8	149.8	26.2

and the variance of the October 1 estimate itself is

$$T - 104.78$$

where T is the *a priori* variance of the year's production.

The second improved system considered provides improvement on the United States alone, by providing the same month-by-month accuracies as the current system, but at times one period earlier (two months). Thus, the standard error in August is reduced from 6.6 percent to 3.2 percent, while the standard error in October is reduced from 3.2 percent to 2.3 percent.

The third improved system provides the same improvement as the second on the United States, while providing a 3 percent standard error on October 1 for the rest of the world.

6.4.2 Distribution Benefits—Corn

The benefits model produces value functions for the United States and the rest of the world whose coefficients are given in Table 6.7. When these are applied to the various information systems described above, benefits are determined as given in Table 6.8.

Here, the potential benefits are substantial, both to the United States and to the rest of the world.

Table 6.7 Value Coefficients Relative to Zero Variance—Corn

Value to \ Forecast Variance of		Coefficients* of Value Function, ($ million/year)/(millions of tons)^2																	
		United States												Rest of the World					
U.S.	Domestic	-.43	-.43	-.44	-.45	-.46	-.47	-.52	-.56	-.63	-.76	-1.05	-2.00	-.13	-.16	-.19	-.25	-.23	.45
	Trade	-.19	-.19	-.20	-.21	-.22	-.24	-.24	-.31	-.41	-.52	-.61	-.24	.21	.24	.28	.32	.17	-.99
	Total	-.61	-.63	-.64	-.65	-.68	-.70	-.76	-.88	-1.04	-1.28	-1.66	-2.24	.08	.08	.09	.07	-.06	-.54
R.O.W.	Domestic	0	0	0	-.01	-.02	-.03	-.01	-.04	-.09	-.14	-.13	.48	-.43	-.50	-.60	-.76	-1.24	-3.31
	Trade	.18	.18	.19	.20	.21	.23	.23	.31	.40	.51	.59	.20	-.21	-.24	-.29	-.32	-.18	1.02
	Total	.18	.18	.19	.19	.20	.20	.22	.26	.31	.37	.46	.67	-.64	-.74	-.88	-1.09	-1.41	-2.28
World	Total	-.43	-.44	-.45	-.46	-.48	-.50	-.54	-.62	-.73	-.91	-1.20	-1.57	-.57	-.66	-.80	-1.02	-1.47	-2.82

*Value obtained by scalar product of coefficient vector with forecast variance vector.

Table 6.8 Distribution Benefits—Corn, millions of 1975 $ per year

	United States			Rest of the World		
	Domestic	Trade	Total	Within	Trade	Total
Case 1	61.4	-14.2	47.2	438.2	17.7	455.9
Case 2	4.2	3.4	8.7	1.6	-4.0	-2.4
Case 3	20.0	83.5	104.6	666.2	-83.0	581.1

7.0 SMALL GRAINS

7.1 Summary of Small Grains Market Factors

"Small grains" is usually taken to include wheat, oats, rye, and barley. For our present purposes, we include oats, rye, and barley alone.

Oats are cultivated mainly in the cooler areas of the world. Cool damp summers are favorable; oats are grown widely in Norway, Northern Ireland, and Scotland, the last having over 50 percent of its planted farm acreage in oats. Oats are not winter hardy and are usually planted in the spring. Some fall oats are sown in France, the pacific coast of the U.S., Ohio and Potomac River areas. Where oats are grown outside North America and Europe, they are mostly fall sown. Oats are the most important feed grain in Canada and number two in the U.S. In Canada, oats compete with barley; in the U.S., with corn. About three percent of the oat crop is used for human food. Industry uses some in plastics, rubber and lubricating oil. Most oats harvested in the U.S. are consumed on the farms where produced. About 25 percent of all U.S. acreage planted in oats is used for silage or forage rather than grain. A farmer receives less return per acre for oats than for corn. Therefore, most oats are used as feed or in crop rotation. On the market, the price of oats is greatly influenced by the price of corn, since they are fairly interchangeable for feed purposes. The U.S.S.R, has become recently the number one producer of oats, followed by the U.S., Canada, West Germany, and Poland. World trade is small due to the bulkiness and weight of oats, which results in high transportation costs. Scientific advances have nearly doubled yields per acre since World War II.

Rye is also grown mainly in the cooler areas of the world. It is the most winter hardy grain. Rye is of minor importance in the U.S., but is used in many parts of the world as a bread grain. Wheat is now taking its place for this purpose in more affluent areas. Most of the world's rye growing is done in Europe (up to 95 percent). Most European production is in Russia, with Poland number two. Also producing are Finland, northern Germany, Denmark, Belgium, France, the Netherlands, and the Scandinavian countries. Canada and Austraila also grow some rye, Canada mainly for its own use. Argentina grows a high protein rye, suitable for baking. Much of it is used for pasturing. Rye is the only other grain besides wheat which can be baked into a loaf. It contains about 1/3 less protein than wheat and less fat. Soft ryes have as little as six percent protein; hard ryes may be over ten percent. Rye is useful for feed, particularly for hogs, but is considered inferior to barley, corn, and oats for that purpose. It is fed in northwestern Europe to bacon hogs, dairy cows, and baby beef steers. Principal exporters are the U.S.S.R., Canada, Poland, the U.S., and the Netherlands. Largest importers are West Germany, the Netherlands, Norway, Sweden and Japan.

Barley is one of the most important feed grains. Most barley is grown in Europe. The U.S.S.R. grows about 25-30 percent of the world total; western Europe grows about 35 percent of the total. North American grows about 16-18 percent. India, Turkey, and South Korea are the largest Asian growers. Barley has been grown widely as a staple food, although it has declined in importance, being replaced by wheat, rye, and rice. It is chiefly used today as livestock feed, and for malting, brewing and distilling. Exporters include France, Canada, U.S.S.R., U.S., and Argentina. The largest importers are West Germany, Italy, Japan, Poland and Belgium.

Treating these three small grains as an aggregate, the crop year can be taken to begin July 1, both in the United States and in the rest of the world.

7.2 Method of Applying Benefits Model to Small Grains Distribution

Three state variables are used in the dynamic programming procedure. Because so much of the production and consumption is in the rest of the world, two state variables refer to the rest of the world, and one to the United States. The United States is the exporting unit and the rest of the world is the importing unit.

The crop year is taken to begin July 1 in both the United States and the rest of the world, and six periods of two-month duration are used.

7.3 Current Information System—Small Grains

7.3.1 United States

Estimates of production of rye, oats, and barley in the United States are published by the USDA Crop Reporting Board in July, August, September, and December. Table 7.1 gives the July, September, December, and final estimates, aggregated over the three grains. We have converted the USDA figures from bushels to metric tons.

Processing these data as described in Chapter 2, we obtain the forecast variance vector

$$\Sigma^2_{US} = (5.90,\ 0.946,\ 0,\ 0.218,\ 0,\ 0.34)\ ,$$

which indicates a total variance of 7.4 or a standard deviation of about 13 percent.

7.3.2 Rest of the World

Just as in the case of corn, the best public source of production estimates during the crop year for small grains is the <u>Grain Bulletin</u> of the Commonwealth Secretariat. As with corn, the data are far from complete.

Table 7.1 United States Small Grains* Production Estimates Published by USDA Crop Reporting Board, millions of metric tons

Crop Year	July	Sept	Nov	Final
1960-1	31.77	32.46	32.25	32.18
1961-2	26.90	27.81	28.45	28.39
1962-3	28.42	29.62	29.74	29.28
1963-4	27.19	27.73	27.91	27.46
1964-5	25.81	25.95	25.90	24.98
1965-6	25.40	28.39	27.71	26.79
1966-7	25.72	24.60	23.74	23.89
1967-8	23.00	23.57	23.01	23.32
1968-9	25.85	27.12	26.90	27.47
1969-0	26.54	27.28	27.53	27.99
1970-1	27.75	26.29	26.74	26.94
1971-2	26.54	27.41	27.07	27.17
1972-3	22.20	22.78	22.03	21.95
1973-4	22.62	22.17	21.29	21.30
1974-5	20.53	19.35	18.47	20.31

*Oats, Rye, and Barley

ECON has projected published estimates for those few countries regularly included to world scale forecasts. The projections for rye are based on figures for West Germany and Poland. The projections for oats and barley are based on figures for France, United Kingdom, and Canada. The resulting "forecasts" of aggregate production in metric tons are given in Table 7.2. In Table 7.3, the final estimates from the Foreign Agricultural Service of the USDA are adjoined, together with a pre-season "forecast" obtained for each year by extrapolation of the preceding year's final estimates. Also included in this table are the "error" means and r.m.s. errors of each period's forecast, normalized to the scale of the 1974-5 crop.

As in the case of corn, the extrapolated "forecast" appears to be very good, certainly better than any of the estimates based on data published during the crop year. The r.m.s. "error" of the extrapolated forecast is 6.2 percent, while the r.m.s. "error" of the other estimates is near 9 percent, with no evident improvement with time! As before, we must conclude that the final estimates are not closely related to the actual system being observed. The total variability of the system can not be conclusively estimated from these data, but we can calculate all but two of the variances of period-to-period forecast differences. These are

Table 7.2 Small Grains* Production Estimates, Projected to World Scale (Excluding U.S.) from Grain Bulletin Figures for West Germany, Poland, France, United Kingdom, and Canada, millions of metric tons

Crop Year	Jul	Sep	Nov	Jan	Mar
1960-61	208.18	152.37	152.66	147.90	149.27
1961-62	140.28	112.47	114.25	119.49	121.38
1962-63	109.93	129.84	140.89	141.78	140.81
1963-64	173.21	142.65	155.18	156.72	188.01
1964-65	139.47	126.85	139.70	148.65	143.35
1965-66	144.18	148.02	146.79	153.68	153.61
1966-67	164.33	155.74	153.49	158.88	159.07
1967-68	175.84	169.75	174.92	177.63	178.02
1968-69		177.20	176.15	176.90	176.78
1969-70	180.14	184.69	184.92	182.79	180.85
1970-71	172.50	172.36	167.53	168.21	164.12
1971-72	185.81	206.91	220.83	225.05	
1972-73	177.29	183.01	193.57	203.08	196.23
1973-74		199.38	192.34	201.69	191.13
1974-75		180.48	174.54	174.20	174.84

*Rye, oats, and barley.

Table 7.3 "Forecast" Matrix for Rest of the World Small Grains*
Production, millions of metric tons

Crop Year	F_0 (Extrapolated)	F_7 (Jul)	F_8 (Sep)	F_9 (Nov)	F_{10} (Jan)	F_{11} (Mar)	Final (May)
1960-1	141.7	141.7	152.4	152.7	147.9	149.3	141.7
1961-2	141.7	140.3	112.5	114.3	119.5	121.4	132.5
1962-3	123.3	109.9	129.8	140.9	141.8	140.8	132.7
1963-4	126.6	173.2	142.7	155.2	156.7	158.0	142.8
1964-5	138.3	139.5	126.9	139.7	148.7	143.4	148.1
1965-6	146.5	144.2	148.0	146.8	153.7	153.6	149.9
1966-7	151.1	164.3	155.7	153.5	158.9	159.1	157.1
1967-8	157.3	175.8	169.8	174.9	177.6	178.0	162.9
1968-9	163.6	163.6	177.2	176.2	176.9	176.8	175.5
1969-0	172.8	180.1	184.7	184.9	182.8	180.9	174.7
1970-1	178.2	172.5	172.4	167.5	168.2	164.1	174.8
1971-2	181.8	185.8	206.9	220.8	225.1	225.1	199.5
1972-3	192.3	177.3	183.0	193.6	203.1	196.2	198.0
1973-4	199.4	199.4	199.4	192.3	201.7	191.1	218.6
1974-5	210.4	210.4	180.5	174.5	174.2	174.8	221.6
Mean "Error"	-8.7	-3.4	-7.1	-2.4	1.8	0	-0.9
Mean Squared "Error"	190.7	431.2	365.8	429.2	375.2	410.5	407.3
R.M.S."Error"	13.8	20.8	19.1	20.7	19.4	20.3	20.2

*Rye, Oats, and Barley

$$\Sigma^2_{ROW} = (0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \sigma^2_6,\ 517.0,\ 87.7,\ 35.5,\ 13.7,\ \sigma^2_{11})\ .$$

If the final estimate were taken as truth, we would have $\sigma^2_{11} = 320.0$, leaving a residual standard error in May of 17.9 or 8 percent. In fact, the residual error is probably less—we assume 6 percent, so that $\sigma^2_{11} = (.06 \times 221.6)^2 = 176.8$. Using T to denote the total variance of the system, we can write

$$\Sigma^2_{ROW} = (0,\ 0,\ 0,\ 0,\ 0,\ 0,\ T\text{-}830.7,\ 517.0,\ 87.7,\ 35.5,\ 13.7,\ 176.8)\ .$$

7.4 Results—Small Grains Distribution

The demand parameters, transportation costs, and production averages used in the calculations of this section are given in Table 7.4.

The variance vectors used to describe the information systems are given in Table 7.5. Three possible improved systems are compared with the current system.

The first, improvement Case 1, does not change the quality of information on the United States, and provides an estimate with standard error six percent on the rest of the world's production at the start of the crop year,

Table 7.4 Small Grains Data other than on Information Systems					
	Demand Parameters			Transportation Cost from United States, $/ton	Mean Annual Production, millions of tons
	Elasticity	Mean Price, $/ton	Mean Consumption, millions of tons/year		
United States	-.6	140	18.53	0	20
Rest of World	-.6	148	221.47	8	220

Table 7.5 Forecast Variance Vectors Describing Information Systems—Small Grains, (millions of metric tons)^2

	Current System	Improvements		
		Case 1	Case 2	Case 3
United States	5.90	5.90	7.03	7.03
	0.95	0.95	0	0
	0	0	0	0
	0.22	0.22	0	0
	0	0	0	0
	0.34	0.34	0.37	0.37
Rest of World	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	T-830.7	T-176.8	T-830.7	T-44.2
	517.0	0	517.0	0
	87.7	0	87.7	0
	35.5	0	35.5	0
	13.7	0	13.7	0
	176.8	176.8	176.8	44.2

July 1. With this system, there is no further estimate, but the true production is known the following May 1. Thus, the standard error of the July 1 estimate is

$$.06 \times 221.6 = 13.30 \text{ million tons} .$$

The variance of the residual uncertainty after this estimate is

$$(13.30)^2 = 176.8 \text{ (million tons)}^2 ,$$

and the variance of the July 1 estimate itself is $T - 176.8$ where T is the a prior variance of the year's production.

The second improved system considered provides improvements on the United States alone. It gives a July 1 forecast having standard error of three percent. No further estimate is provided, but the true production is known the following May 1. For this case, the standard error of the July 1 estimate of United States production is

$$.03 \times 20.31 = .609 \text{ million tons} .$$

The variance of the residual uncertainty after this estimate is

$$(.609)^2 = .037 \text{ (million tons)}^2 ,$$

and the variance of the July 1 estimate itself is

$$7.40 - .037 = 7.03 \text{ (million tons)}^2 .$$

Finally, the third improved system provides the same three percent standard error July 1 in both the United States and the rest of the world. Thus, the non-zero elements of the variance vector for the rest of the world are

$$(.03 \times 221.6)^2 = 44.2$$

and

$$T - 44.2 .$$

The coefficients of the loss functions from the benefits model are as given in Table 7.6. When these are applied to the differences of the variance vectors of Table 7.5, the resulting benefits are as given in Table 7.7.

Table 7.6 Value Coefficients Relative to Zero Variance—Small Grains

Coefficients* of Value Function, ($million/year)/(millions of tons)^2

Value to / Forecast Variance on	United States			Rest of World			World
	Domestic	Trade	Total	Domestic	Trade	Total	Total
United States	-3.641	1.580	-2.061	1.939	-1.664	0.275	-1.786
	-4.100	1.677	-2.423	2.096	-1.766	0.329	-2.094
	-4.731	1.710	-3.021	2.207	-1.808	0.399	-2.622
	-5.985	1.902	-4.083	2.505	-2.024	0.481	-3.602
	-8.920	2.715	-6.205	3.496	-2.897	0.598	-5.607
	-20.184	8.115	-12.069	9.103	-8.443	0.660	-11.409
Rest of World	-0.021	0.035	0.014	-0.361	-0.036	-0.397	-0.383
	-0.021	0.036	0.014	-0.366	-0.036	-0.402	-0.388
	-0.021	0.036	0.015	-0.371	-0.037	-0.408	-0.393
	-0.022	0.037	0.015	-0.377	-0.037	-0.415	-0.400
	-0.022	0.037	0.016	-0.385	-0.038	-0.423	-0.408
	-0.023	0.040	0.017	-0.396	-0.041	-0.437	-0.420
	-0.024	0.043	0.019	-0.428	-0.044	-0.471	-0.453
	-0.030	0.052	0.022	-0.502	-0.053	-0.554	-0.532
	-0.038	0.065	0.026	-0.606	-0.066	-0.671	-0.645
	-0.051	0.082	0.031	-0.765	-0.084	-0.849	-0.818
	-0.073	0.103	0.030	-1.045	-0.105	-1.151	-1.120
	0.045	-0.075	-0.030	-1.858	0.078	-1.781	-1.811

*Value obtained by scalar product of coefficient vector with forecast variance vector

Table 7.7 Distribution Benefits—Rye, Oats, and Barley, millions of 1975 $ per year

	United States			Rest of the World		
	Domestic	Trade	Total	Domestic	Trade	Total
Case 1	5.75	-8.80	-3.05	74.18	8.92	83.10
Case 2	.46	.03	.49	-.06	-.03	-.09
Case 3	-3.02	6.89	3.88	263.82	-7.17	256.65

8.0 SUMMARY AND CONCLUSIONS

There are substantial distribution benefits potentially available to the United States through improved information on worldwide crop production, particularly in the cases of wheat and corn. Table 8.1 collects the United States Benefits derived in this report for corn, potatoes, small grains, soybeans, and sugar. Two cases are presented: an information system providing only modest improvements (six percent standard error at harvest) and only on the rest of the world; and an information system providing a greater improvement, this time on both the United States and the rest of the world. Perhaps this second case represents a reasonable expectation for a thematic mapper system.

The potential for economic benefits lies mainly in information on corn production. This is because corn exports are very large and important to the United States, and there is a large uncertainty in export demand.

For potatoes, the potential is quite limited, at least in terms of the aggregate United States supply as treated in this study. First of all, potatoes are essentially unstorable, so there is not an effective way for the market to respond to better supply information. United States potatoes are distributed only within the United States, so the worldwide markets are not involved.

Table 8.1 Distribution Benefits of Improved Crop Production Information

Crop	United States Benefits, millions of 1975 dollars per year					
	Improvement on ROW only, 6% std. error at harvest			Improvement on US and ROW. Earlier information on US, 3% std. error at harvest in ROW (Thematic Mapper)		
	Domestic	Trade	Total	Domestic	Trade	Total
Corn	61	-14	47	20	84	104
Potatoes	0	0	0	0	0	0
Rye, Oats, Barley	5.8	-8.8	-3.1	-3.0	6.9	3.9
Soybeans	0.2	0.4	0.6	6.2	4.3	10.5
Sugar	0	0	0	0.4	-0.2	0.2
Total	66.95	-22.4	44.6	23.6	95.0	118.6

And finally, the existing uncertainty is not only of production, but of quality, shrinkage and loss as well. Thus it is more difficult for a satellite information system to contribute in this case than in others.

The potential distribution benefits associated with rye, oats, and barley are moderate. Although these grains have some of the characteristics required for effective competition with wheat as a food, the present structure of demand relegates them primarily to the feed grain class. Further, the United States is presently exporting relatively little of these grains, and its production and stocks have been declining.

Soybeans are, of course, extremely important to the United States economy, largely through international trade. However, the current state of information is fairly good, partly because so much of the world's production comes from the United States. Thus, the annual benefit to be expected from a system such as described above (3 percent standard error at harvest) is only about $10 million. However, further improvements would produce additional benefits.

Since the United States is an importer of sugar, it stands to benefit (through the distribution mechanisms) only from improved information on its own crop. Improved information on the foreign crops is actually of negative value to the United States. It is possible that the benefits (or disbenefits) as given in Chapter 4 and the summary table (Table 8.1) are understated. The published data on sugar supply are extremely unreliable, and are difficult to use because of the complexity of the sugar markets. Further, the price histories contain less than usual amounts of information because of such distortions of the free market as price regulation and import quotas, as well as long term trade agreements. Thus, it may be that the current state of world information is much poorer than we have portrayed it. In any case, the potential distribution benefits to the United States of improved information on sugar production are minor.

The analysis of this study has treated the various crops independently; thus, it does not provide an accurate picture of the benefits that would be achieved from simultaneous improvements in information on, say, corn and soybeans. It is likely that the simple sum of the benefits estimated here would underestimate the actual benefits, since the possibility of substitutions provides more opportunities for the efficient use of the distribution mechanisms (trade and storage), provided there is adequate information.

For example, it is possible that improved knowledge of wheat and corn production in Asia could result in more efficient trade of United States soybeans, but such an effect is missed in the analysis performed to date.

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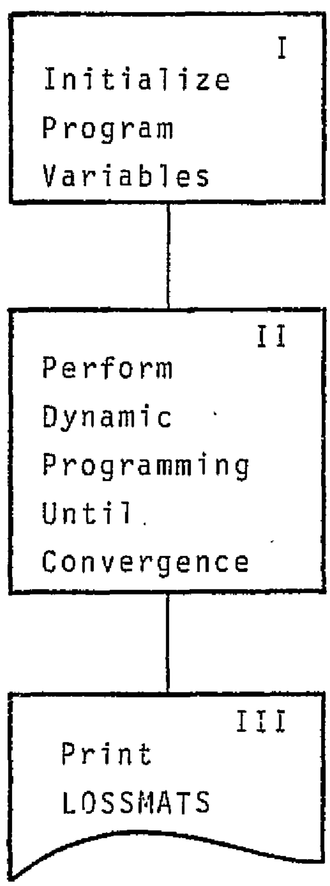
APPENDIX

ALGORITHM FOR CALCULATION OF VALUE FUNCTION COEFFICIENTS

For each commodity, perform this procedure to obtain the LOSSMATS (coefficient matrix for value function):



The Algorithm

Block I Initialize all program variables.

Block II Perform dynamic programming procedure until convergence is achieved. The initial value function is calculated based on a simple heuristic decision rule. This procedure converges on the optimal period-by-period decision rules and simultaneously yields the various components of the value functions of each unit (exporter and importer).

Block III Print the LOSSMATS for each component of the value functions obtained in Block II. These LOSSMATS, when multiplied component-wise by a stochastic variance matrix, will yield the economic loss (relative to no variation at all) of that particular stochastic model.

Procedure Block I

INITIALIZATION

Note: Unit 1 (state variable 1) is the exporter, and unit 2 is the importer. State variable 3 may reflect advance information on either Unit 1 or Unit 2 depending on the setting of the program switch SV3 (=1 or 2).

Step 1 Input.

(a) e_v ($v = 1, 2$) = price elasticities for unit v.

(b) p_v ($v = 1, 2$) = sample price for unit v.

(c) c_v ($v = 1, 2$) = sample annual consumption of unit v.

(d) r = annual discount rate (6%).

(e) m = number of periods per year. One year/m must be the average interval between order and delivery of exports.

(f) τ = transportation cost per unit of exports.

(g) π_v = annual production of unit v.

(h) κ_v ($v = 1, 2$) = presumptive inventory carryover for unit v.

Step 2 Compute Annual Demand Function Parameters.

Exporter's price $\hat{p}_1 = \beta + 2\alpha q$

Importer's price $\hat{p}_2 = \delta + 2\gamma q$

where q = annual consumption.

(a) $\alpha \leftarrow \frac{p_1}{2c_1 e_1}$ = ½ slope of exporter's demand function, expressing price as a function of annual consumption.

(b) $\beta \leftarrow p_1(1 - \frac{1}{e_1})$ = intercept of exporter's demand function.

(c) $\gamma \leftarrow \frac{p_2}{2c_2 e_2}$

(d) $\delta \leftarrow p_2(1 - \frac{1}{e_2})$

Step 3 Data Adjustments for Length of Period.

(a) $\rho \leftarrow (\frac{1}{1+r})^{\frac{1}{m}}$ = discount factor for a single period

(b) $\alpha \leftarrow m\alpha$ = unit 1's periodic demand parameter.

(c) $\gamma \leftarrow m\gamma$ = unit 2's periodic demand parameter.

Step 4 Definition of constant matrices.

$$A = \begin{pmatrix} \alpha & 0 & \alpha \\ 0 & 0 & 0 \\ \alpha & 0 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} \beta \\ 0 \\ \beta \end{pmatrix}$$

$$E = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \gamma & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad F = \begin{pmatrix} \beta \\ \delta \\ -\tau \end{pmatrix}$$

$$C = E - A = \begin{pmatrix} 0 & 0 & -\alpha \\ 0 & \gamma & 0 \\ -\alpha & 0 & 0 \end{pmatrix}$$

$$D = F - B = \begin{pmatrix} 0 \\ \delta \\ -\tau-\beta \end{pmatrix}$$

$$M = \begin{pmatrix} -1 & 0 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Step 5 Exit Block.

Procedure Block II

DYNAMIC PROGRAMMING

Note: When $(n + 1)$ is used as a subscript, it is taken as $1 + \text{mod}_m(n)$. That is, if $n = m$, then $Q_{n+1} \equiv Q_1$.

Step 1 Initialize Period 1 total value function.

(a) $$Q_{1,1} = \begin{pmatrix} \frac{\alpha}{mr} & 0 & 0 \\ 0 & \frac{\gamma}{mr} & 0 \\ 0 & 0 & \frac{\alpha}{mr(1+r)} \end{pmatrix} \text{ if } SV3 = 1, \text{ else}$$

$$\begin{pmatrix} \frac{\alpha}{mr} & 0 & 0 \\ 0 & \frac{\gamma}{mr} & 0 \\ 0 & 0 & \frac{\gamma}{mr(1+r)} \end{pmatrix} \text{ if } SV3 = 2 \text{ .}$$

(b) $$P_{1,1} = \begin{pmatrix} \frac{\beta}{r} \\ \\ \frac{\delta}{r} \\ \\ \frac{\beta}{r(1+r)} \end{pmatrix} \text{ if SV3 = 1, else}$$

$$\begin{pmatrix} \frac{\beta}{r} \\ \\ \frac{\delta}{r} \\ \\ \frac{\beta}{r(1+r)} \end{pmatrix} \text{ if SV3 = 2 .}$$

<u>Step 2</u> $n \leftarrow m$.

Step 3

Let $N = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ if $n \neq m$, else

$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ if $(n = m) \wedge (SV3 = 1)$, else

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ if $(n = m) \wedge (SV3 = 2)$.

Step 4

Select 27 states (grid points)

$S_{ijk}(i = 1, 2, 3;\ j = 1, 2, 3;\ k = 1, 2, 3) =$

$$\mu_n + \sqrt{\tfrac{3}{2}} \times \sigma_n \times \begin{pmatrix} i-2 \\ j-2 \\ k-2 \end{pmatrix}$$

where

(a) μ_n = the grid mean in each dimension. The μ's are calculated based on the assumption of smooth consumption and export rates and some presumptive year-end carryover (minimum stock) level; and

(b) σ_n = the desired standard deviation of the grid in each dimension.

<u>Step 5</u> Quadratic Programming.

For each S_{ijk} in Step 4, find the 1-stage optimal decision vector $\hat{Y}_{ijk}$ by maximizing

$$W = Y^*ZY + Y^*G$$

subject to the constraints

$$Y \geq 0 \;;$$

$$Y_1 + Y_3 \leq S_1 \;; \text{ and}$$

$$Y_2 \leq S_2$$

where

$$Z = E + \rho M^*PM \text{ and}$$

$$G = F + \rho M^*(2Q_{n+1,1}(NS_{ijk} + \Pi_n) + P_{n+1,1}) .$$

Note that Π_n is the period-by-period production input. Thus

$$\Pi_t = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{if } t \neq m, \text{ else}$$

$$\begin{pmatrix} \pi_1 \\ 0 \\ \pi_2 \end{pmatrix} \quad \text{if } (t = m) \wedge (SV3 = 2), \text{ else}$$

$$\begin{pmatrix} 0 \\ \pi_2 \\ \pi_1 \end{pmatrix} \quad \text{if } (t = m) \wedge (SV3 = 1).$$

Step 6

For each S and $\hat{Y}$, compute $T_n(S)$.

$$T_{ijk} = NS_{ijk} + M\hat{Y}_{ijk}.$$

Step 7

For each S, calculate the components of the discounted value functions for each unit. Only four components need be saved because the others may be calculated by taking simple differences.

(a) Total value (both units) =

$$\left\{\rho T^*(P_{n+1,1} + Q_{n+1,1}T) + Y^*(F + EY)\right\}$$

(b) Unit 1's internal consumption =

$$\left\{\rho T^*(P_{n+1,2} + Q_{n+1,2}T) + y_1(b_1 + a_{1,1}y_1)\right\}$$

(c) Unit 1's export value =

$$\left\{\rho T^*(P_{n+1,3} + Q_{n+1,3}T) + y_3(b_1 + 2a_{3,1}y_1)\right\}$$

(d) Unit 2's internal consumption =

$$\left\{\rho T^*(P_{n+1,4} + Q_{n+1,4}T) + y_2(f_2 + e_{2,2}y_2)\right\}$$

<u>Step 8</u>

Least-squares-fit. Do (4) least squares fits of the quadratic form

$$S^*Q_{n,i}S + S^*P_{n,i} + R_{n,i} (i = 1, 2, 3, 4)$$

to the 4 sets of values computed in Step 7. Note that if all the maximizers in Step 5 were unconstrained, the least squares fits would have zero residual.

This step generates the new period n value function parameters

$Q_{n,i}$, $P_{n,i}$ and $R_{n,i}$ $(i = 1, 2, 3, 4)$. Note also that R is not needed for further calculations and therefore may be discarded.

Step 9

$n \leftarrow n - 1$. If $n \geq 1$ then go to Step 3,

Step 10

Check for convergence: if $Q_{n,i}$ or $P_{n,i}$ ($n = 1, 2, \ldots, m$; $i = 1, 2, 3, 4$) have changed significantly since the last iteration, go to Step 2.

Step 11 Exit Block.

Note: this procedure usually converged to within $\pm$ 0.1% in 7 iterations and at most in 10 iterations.

Output Block III

The LOSSMATS are based on the matrices

$$Q_{n,i} (n = 1, 2, \ldots, m; i = 1, 2, 3, 4).$$

Because the loss terms are of the form

$$\Phi_n^{\;*} Q_{n+1,i} \Phi_n ,$$

The LOSSMATS consist of the diagonal elements of the Q matrices rotated left by 1 period.
For example, the exporter's internal LOSSMAT is comprised of the diagonal elements from $Q_{n,2}$ (n = 2, 3, ..., m, 1).

(a) Exporter's internal: $Q_{n,2}$

exports: $Q_{n,3}$

total: $Q_{n,2} + Q_{n,3}$

(b) Importer's internal: $Q_{n,4}$

exports: $Q_{n,1} - (Q_{n,2} + Q_{n,3} + Q_{n,4})$

total: $Q_{n,1} - (Q_{n,2} + Q_{n,3})$

(c) Whole world exports: $Q_{n,1} - (Q_{n,2} + Q_{n,4})$

total: $Q_{n,1}$.